

What the Standard Model may not want us to know

On the search for a nonperturbative regulator for chiral gauge theories

Dorota Grabowska, D.B.K., arXiv:1511.03649, to appear in PRL



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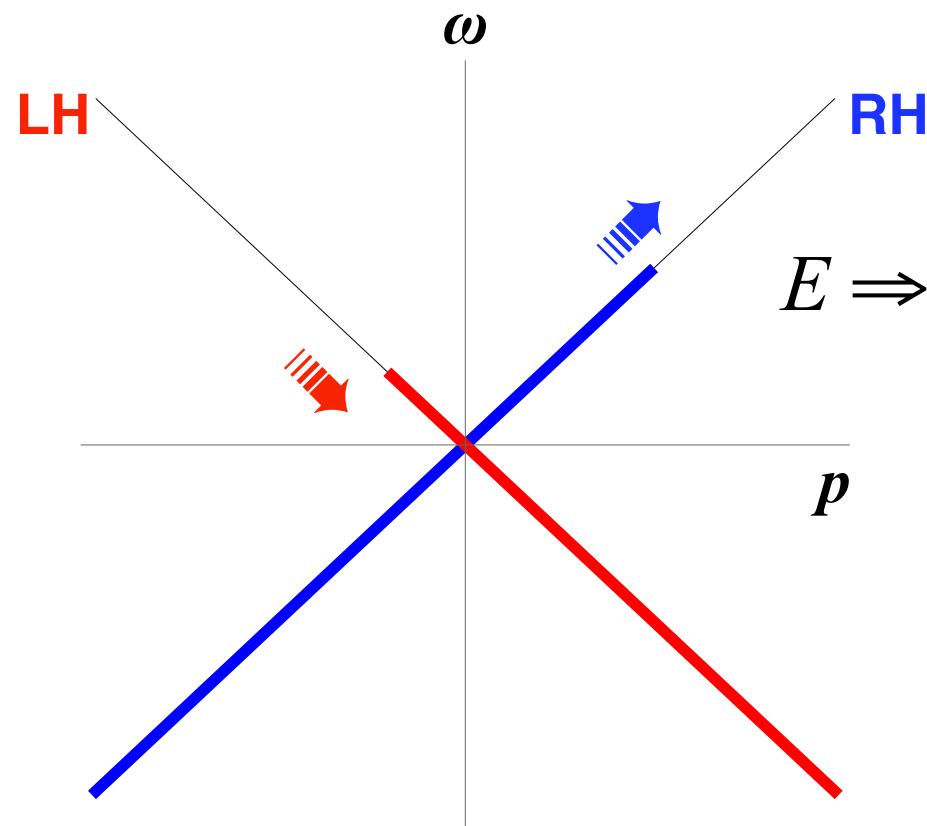
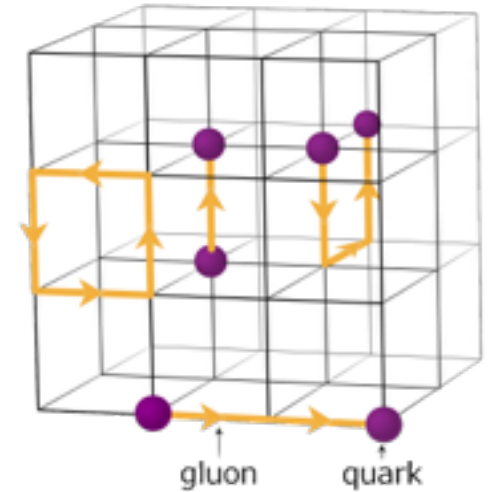
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Or might something be missing from the Standard Model?

What are the issues for chiral symmetries on the lattice?

The key is the anomaly.



$$d=(1+1): \quad \partial_\mu j_5^\mu = \frac{qE}{\pi}$$

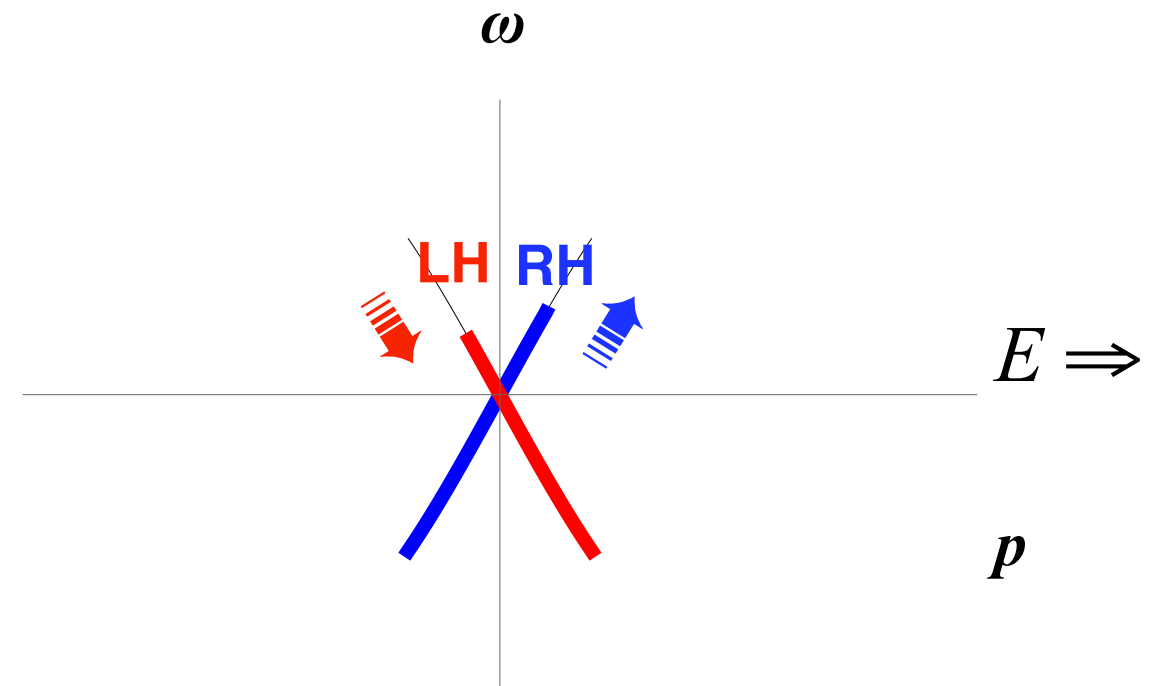
quantum violation of a classical symmetry



In the continuum, the Dirac sea is filled...but is a Hilbert Hotel which always has room for more

Not so on the lattice:

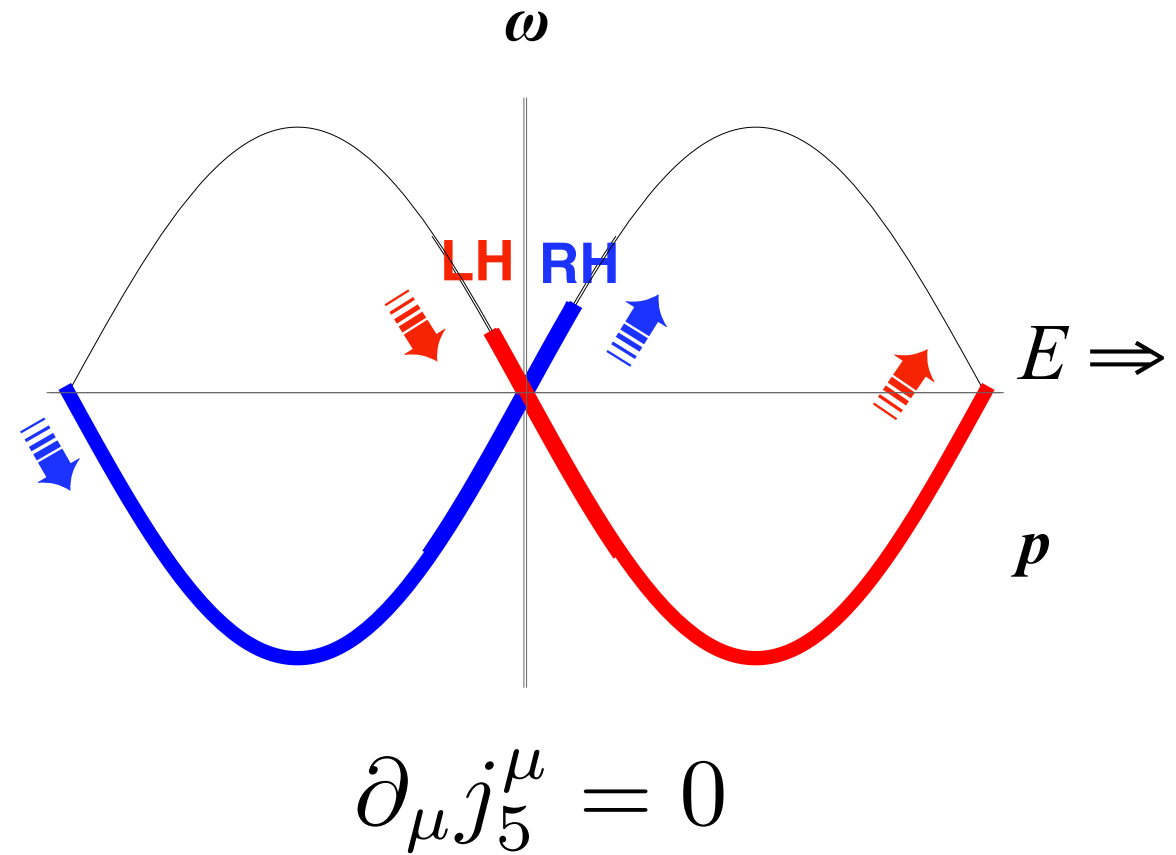
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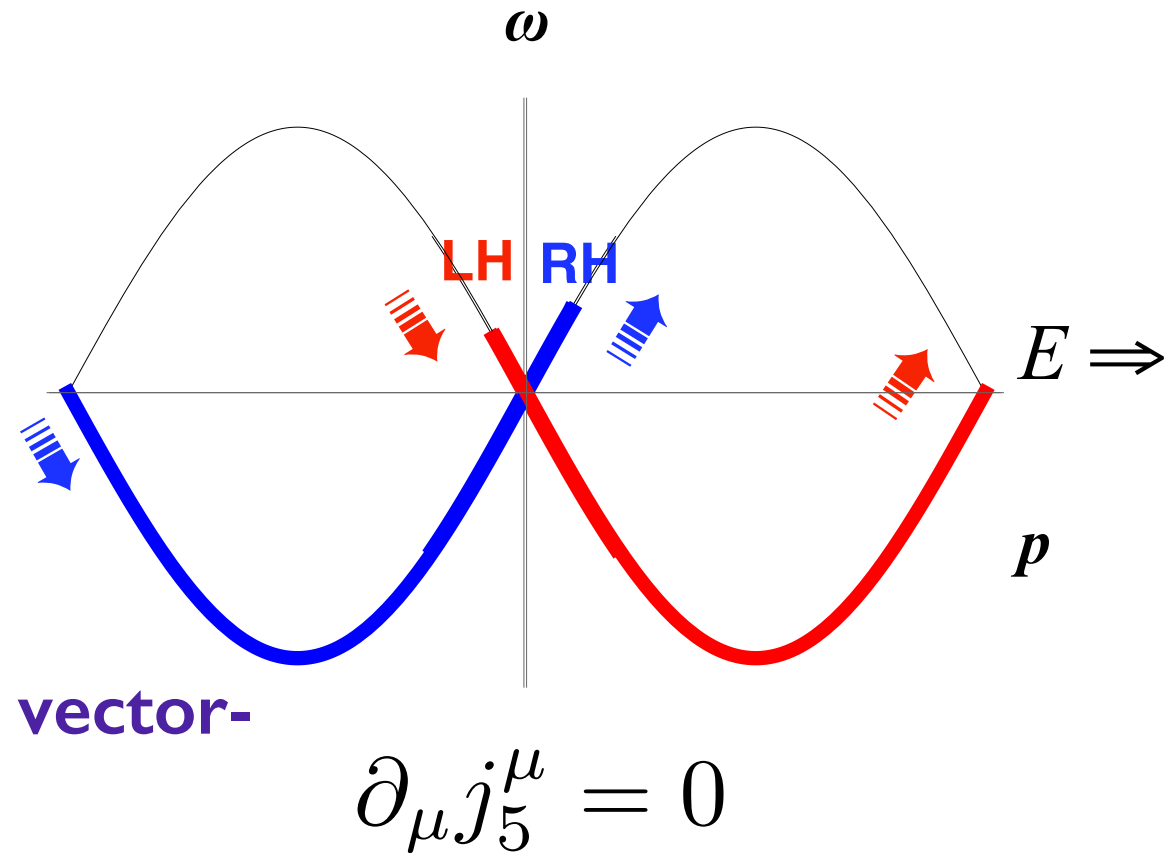


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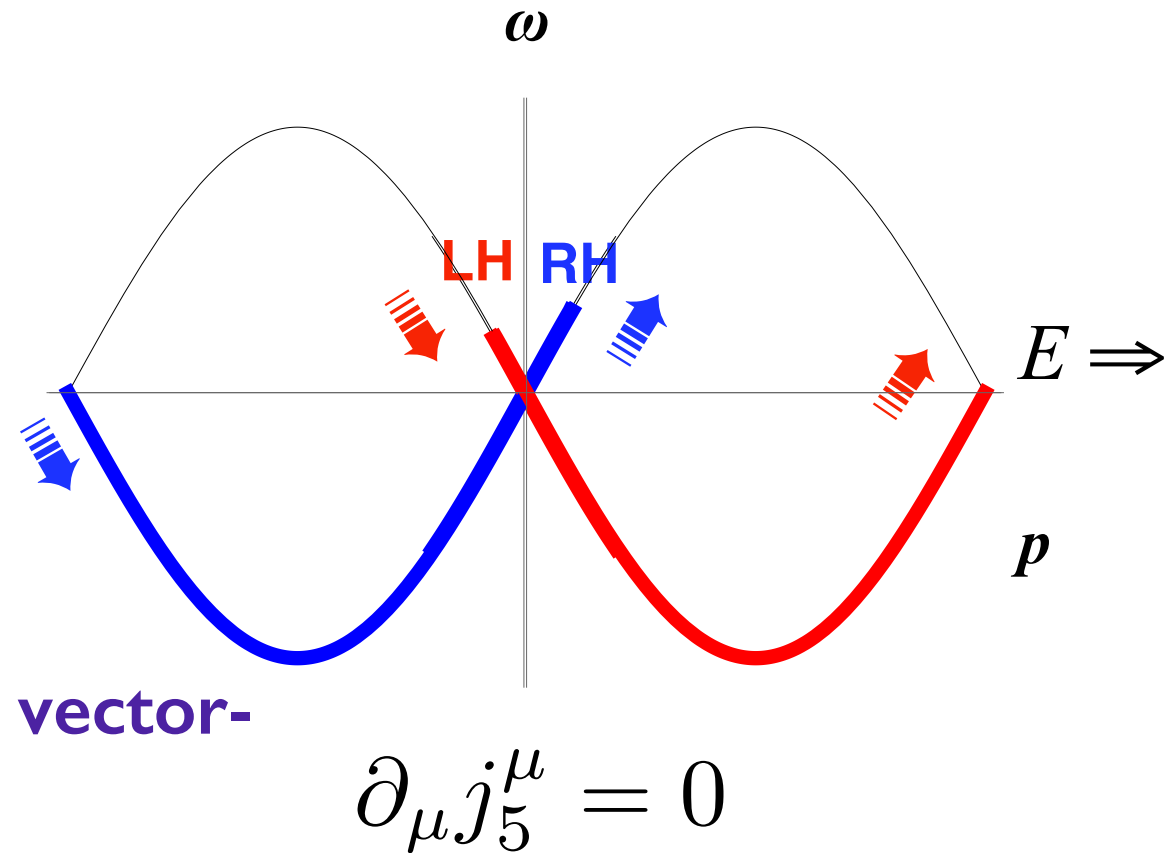


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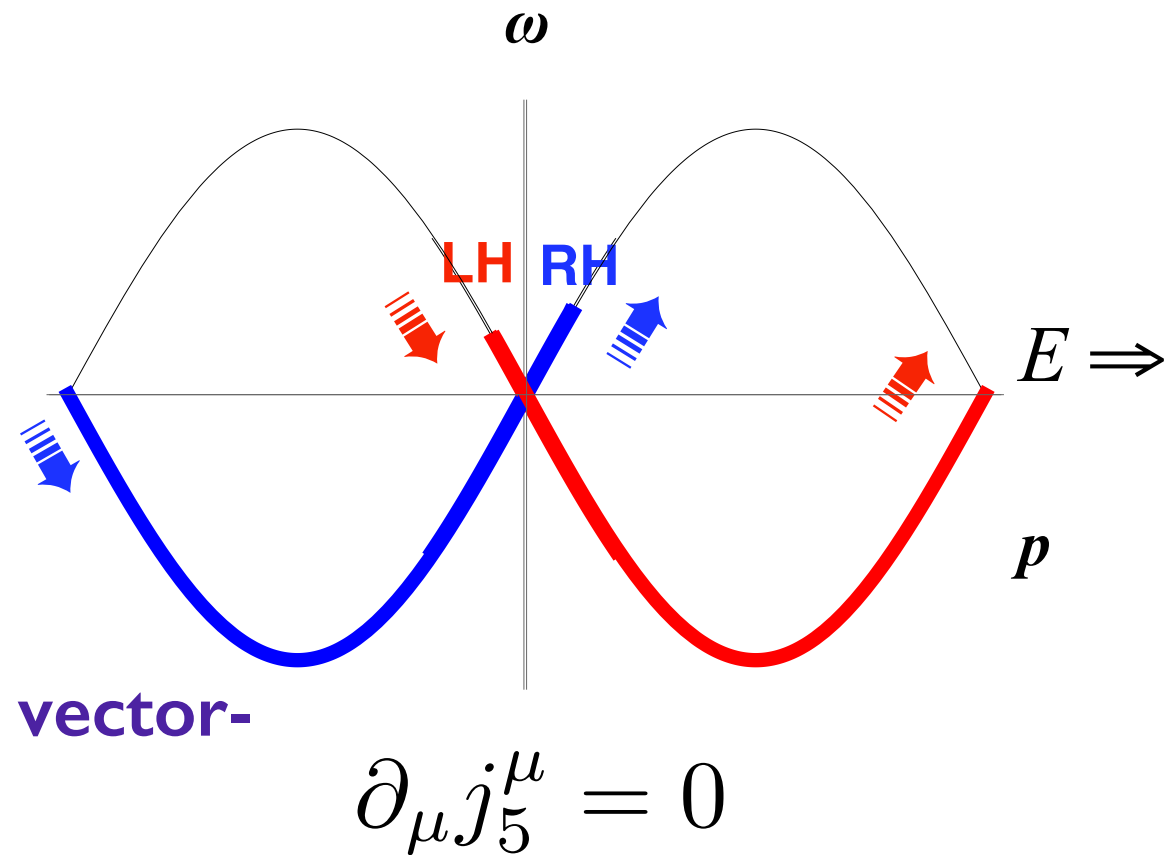


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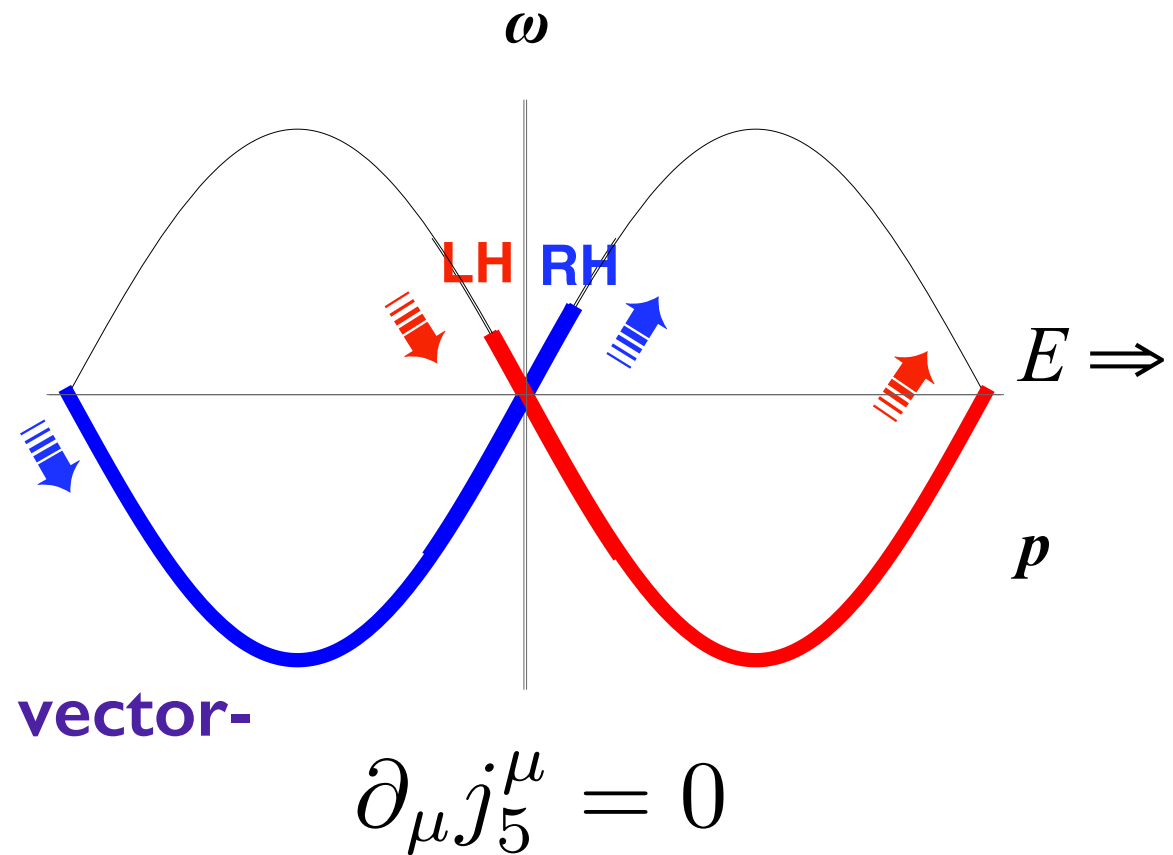


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- ...these unwanted “mirror fermions” are not seen in the SM.
- The mirror fermions can be given a large mass and decoupled in a vector gauge theory, but this breaks gauge symmetry in a chiral gauge theory

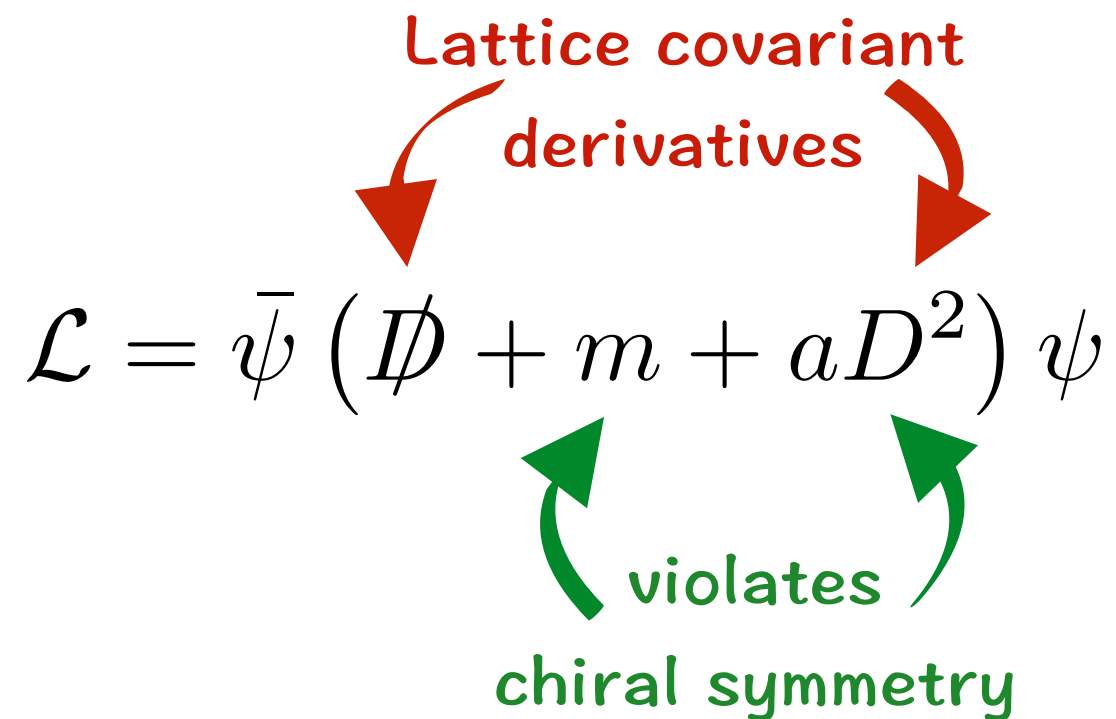


How lattice QCD with Wilson fermions reproduces the $U(1)_A$ anomaly:

Lattice covariant
derivatives

$$\mathcal{L} = \bar{\psi} (\not{D} + m + aD^2) \psi$$

violates
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- Wilson fermions eliminate doublers by giving them a big mass
- This explicitly breaks the (global) chiral flavor symmetries of QCD
- Correct anomalous Ward identities are recovered in the continuum through fine tuning

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For chiral gauge theories, a devil's choice:

- keep the mirror fermions (not a chiral theory!)
- eliminate the mirror fermions (breaks the gauge symmetry!)

In lattice QCD (or any vector like theory):
give the mirror matter fields a large mass
and decouple them



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For a chiral gauge theory, break
the gauge symmetry to
eliminate the mirror matter?
*(...and try to restore it in the
continuum limit?)*

Bock, Golterman, Shamir 1998, 2004

Breaking gauge symmetry opens
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A third option? exotic strong dynamics confine
the mirror fermions*

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4-fermion interactions (Eichten-Preskill),
strong Yukawa couplings (Smit-Swift; Poppitz-Shang) . . .



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4-fermion interactions (Eichten-Preskill),
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Numerous attempts in the literature, but:
Every case analyzed: the mirrors do not decouple,
or the whole theory becomes noninteracting...
(Golterman-Petcher-Rivas-Smit; Chen-Giedt-Poppitz)



This talk:

the mirror fermions do not become heavy, but decouple in a gauge invariant way...

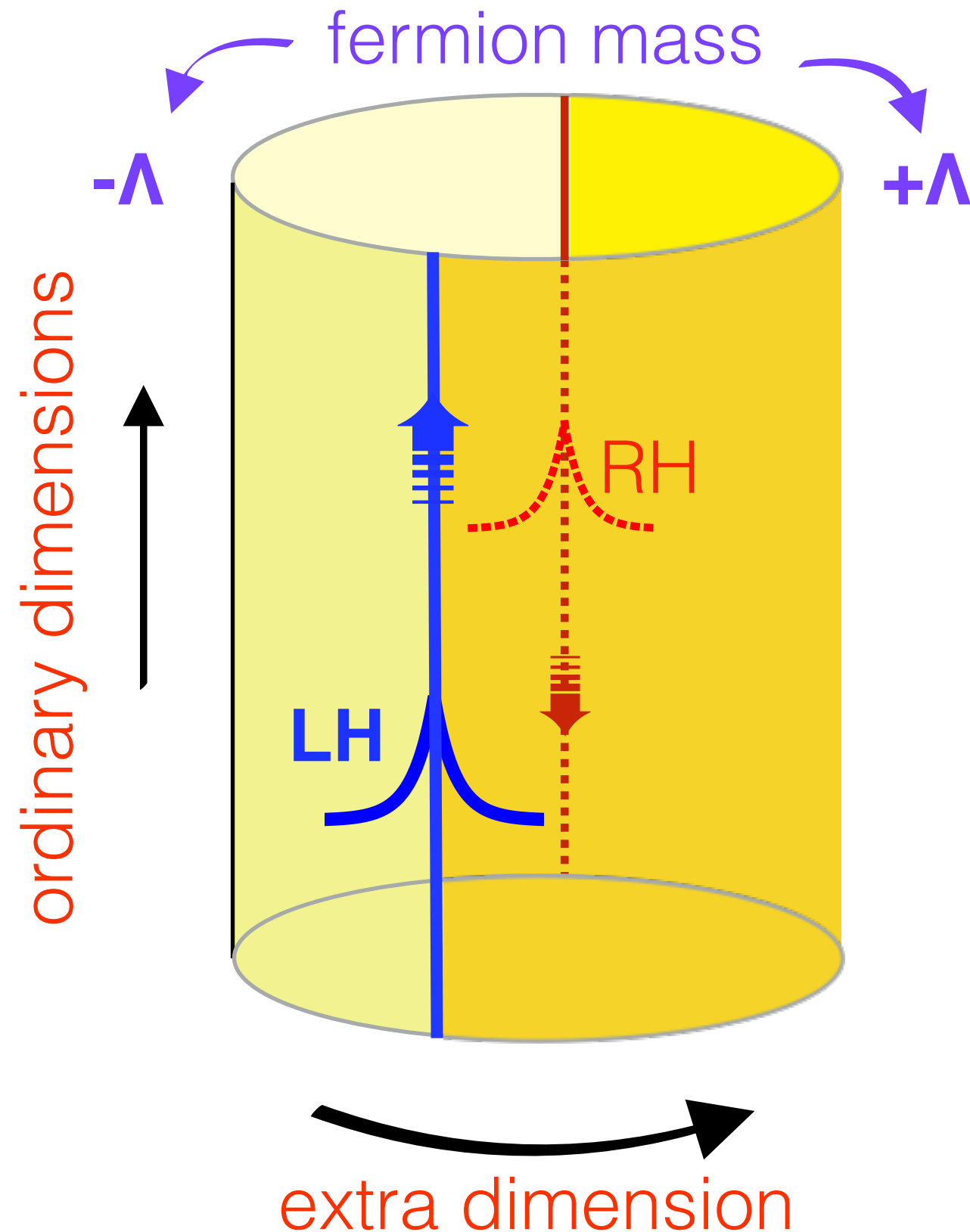
...but not entirely: still have potentially interesting nonlocal interactions through gauge topology

Starting point: domain wall fermions (aka: topological insulators)

Start with an easier problem:

realizing global chiral symmetries in QCD (DBK, 1992)

- Dirac field in compact 5th dimension
- Gauge fields constant in x_5
- Fermion mass flips sign as you go around x_5
- Find localized massless LH, RH chiral modes stuck to mass defects
- At low energy and $L_5 \Rightarrow \infty$, gauge fields see a massless Dirac fermion
- $U(1)_A$ anomaly: massive bulk fermions do not quite decouple, communicate chiral charge flow between domain walls



Integrating out massive fermion modes from the bulk leads to a non-decoupling term: the Chern-Simons action

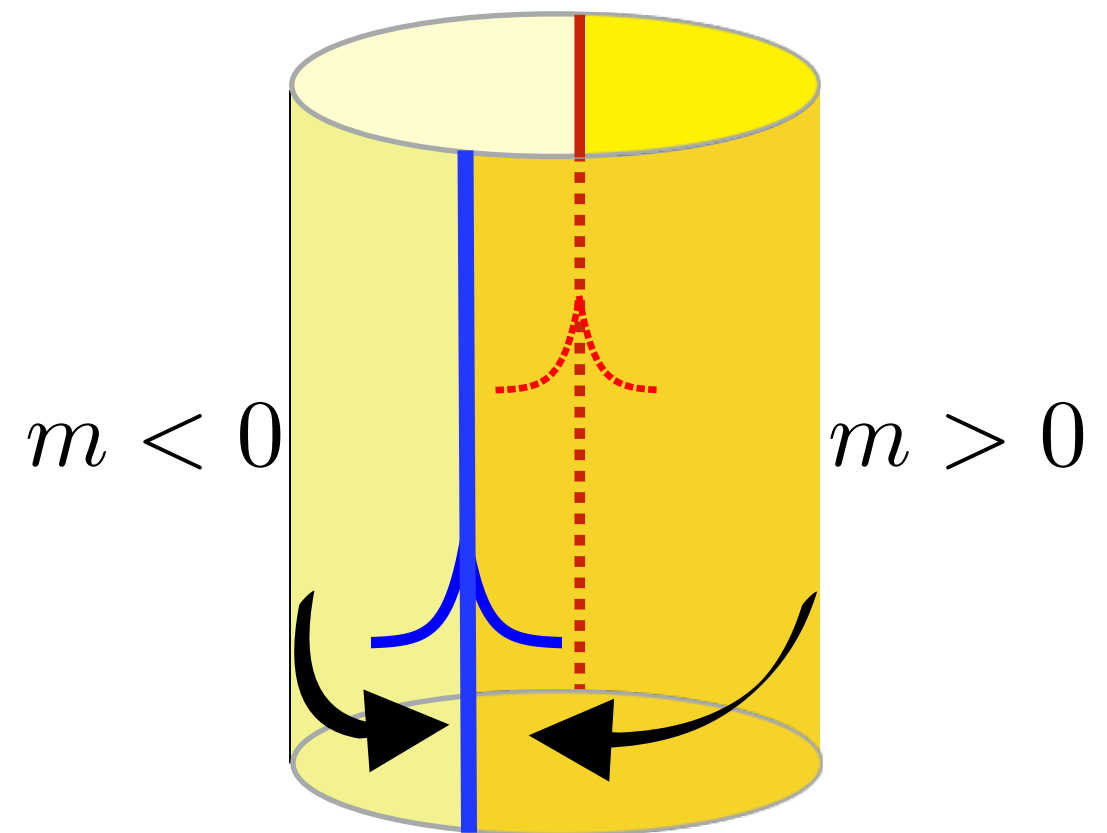
Sensitive to sign of m , no decoupling as $m \rightarrow \infty$

$$S_{\text{eff}} \propto \frac{m}{|m|} \epsilon_{abcde} \text{Tr} (A_a \partial_b A_c \partial_d A_e + \dots)$$

Yields current that diverges where the massless fermions live:

$$\partial_\mu j_5^\mu \propto \frac{m}{|m|} \epsilon_{5bcde} \text{Tr} F_{bc} F_{de}$$

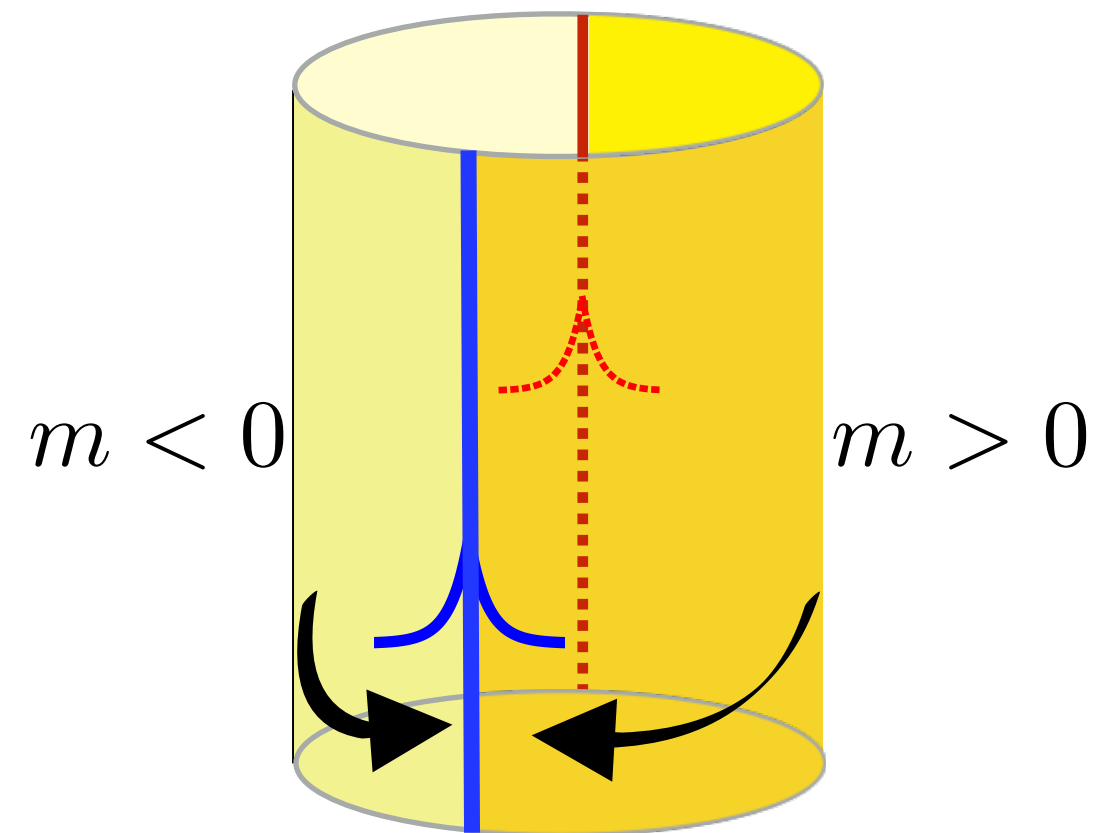
All chiral symmetry breaking due to bulk fermion mass decouples as $m \rightarrow \infty$ *except* for the anomaly...no fine-tuning needed



System is stable against radiative corrections because number of chiral zero modes is determined by **topology** (hence *topological* insulator)

The topology is not in coordinate space, but is a property of the bulk fermion dispersion relation in momentum space

(Golterman, Jansen, DBK, 1992)

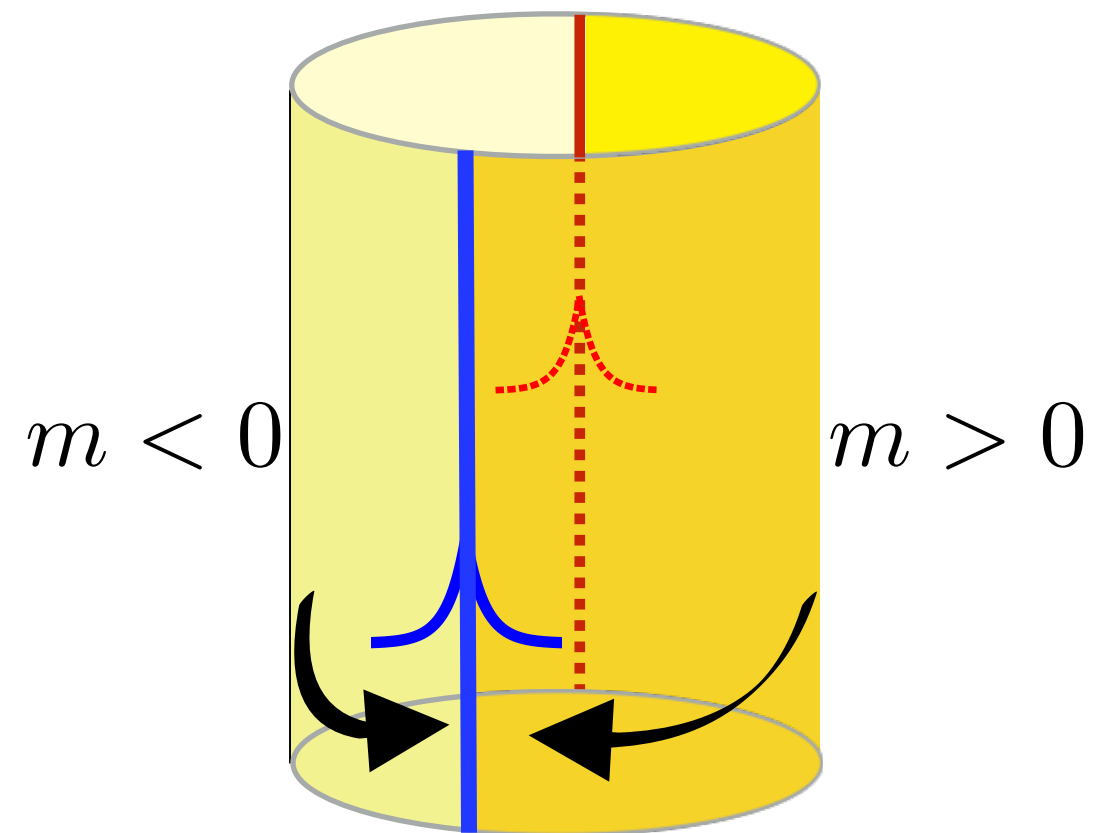


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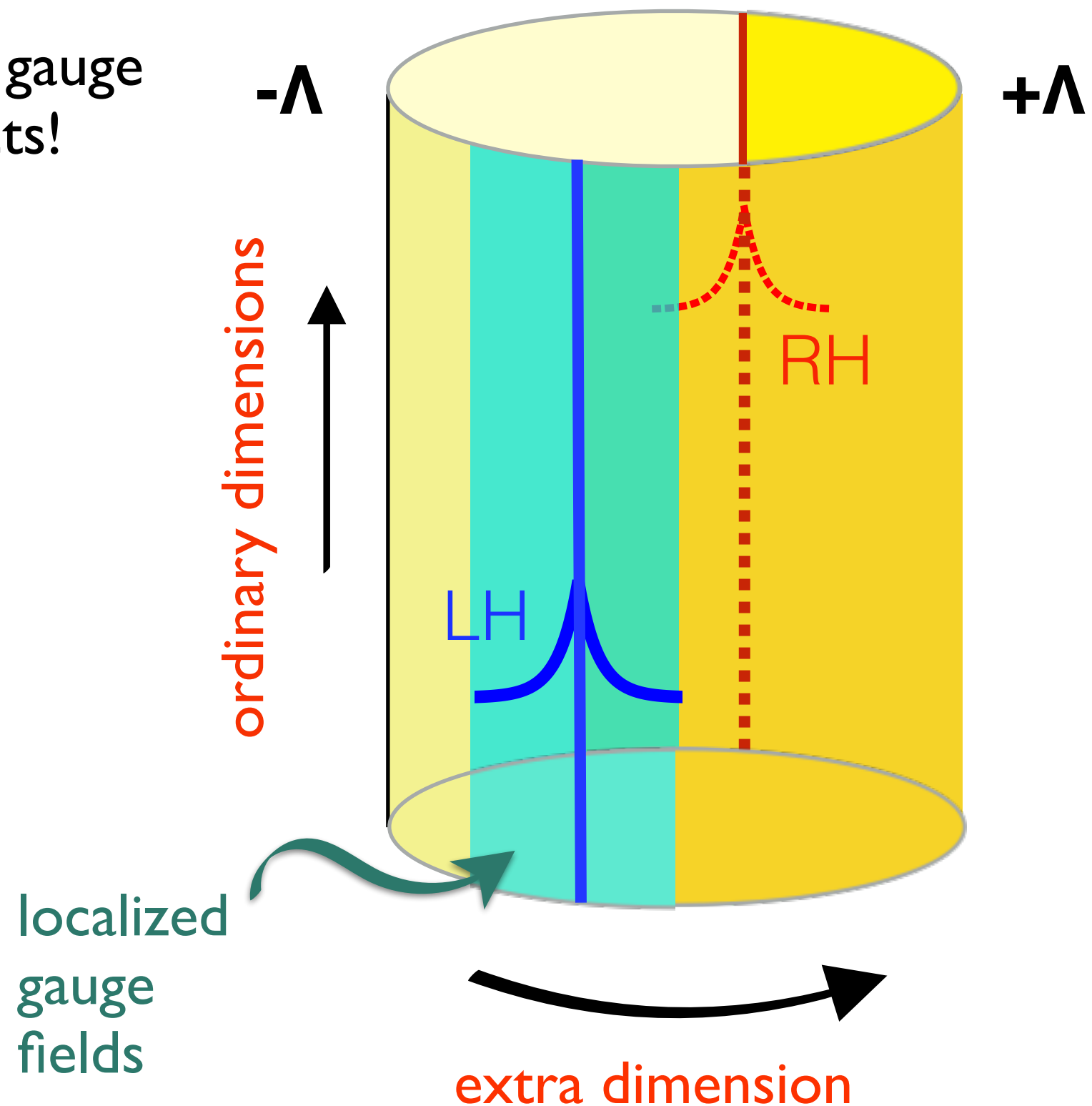
The simple model of 2 bulk fermions with opposite sign for the mass (so that the CS term cancels) is the Kane-Mele Quantum Spin Hall Effect (2004)



Old attempts to use domain wall fermions for chiral gauge theory

Obvious solution: localize the gauge fields around one of the defects!

- fermions = gauged
- mirrors = neutral



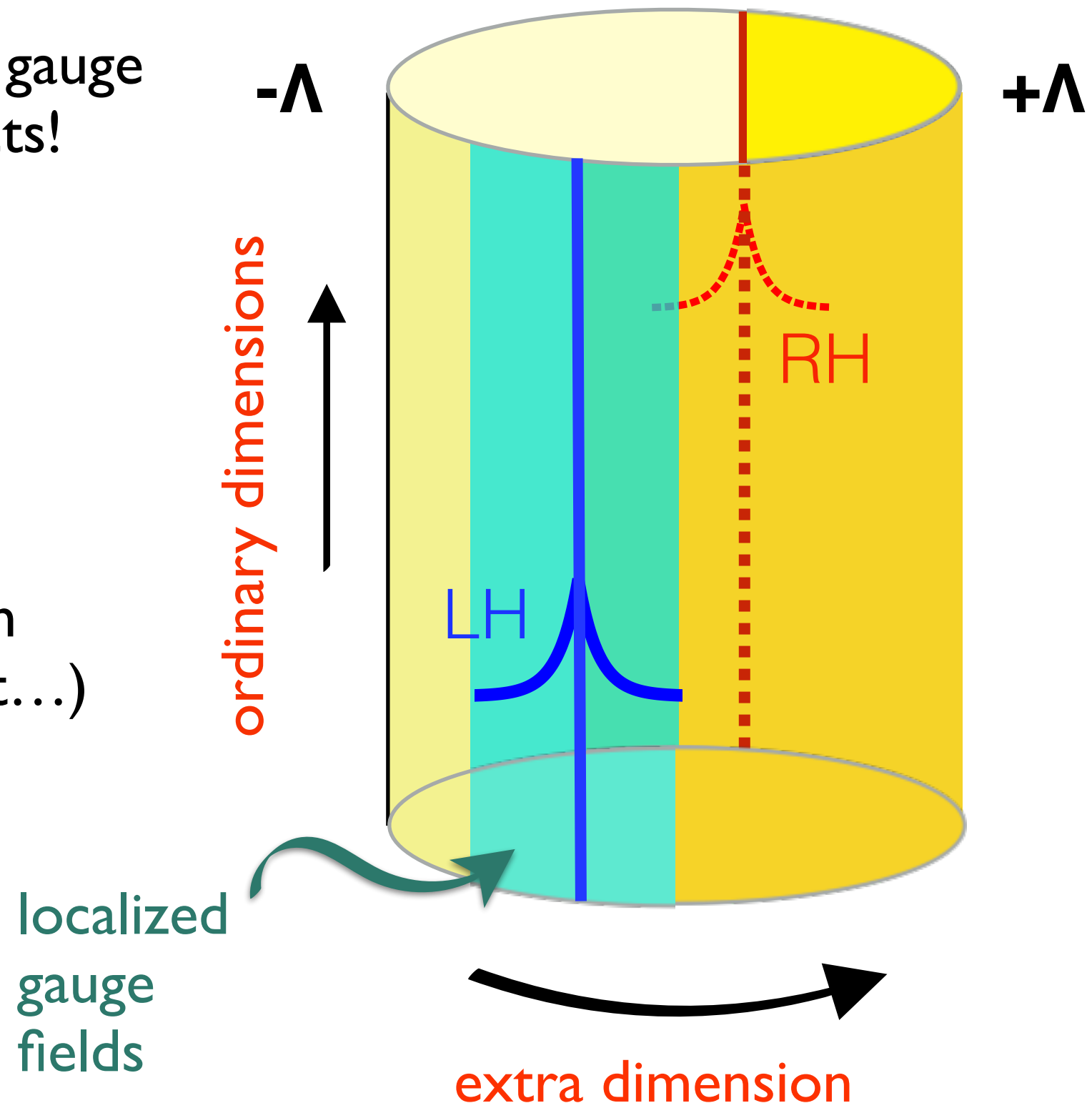
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This requires:

- 5d gauge fields
- phase transition away from defect (Higgs, confinement...)



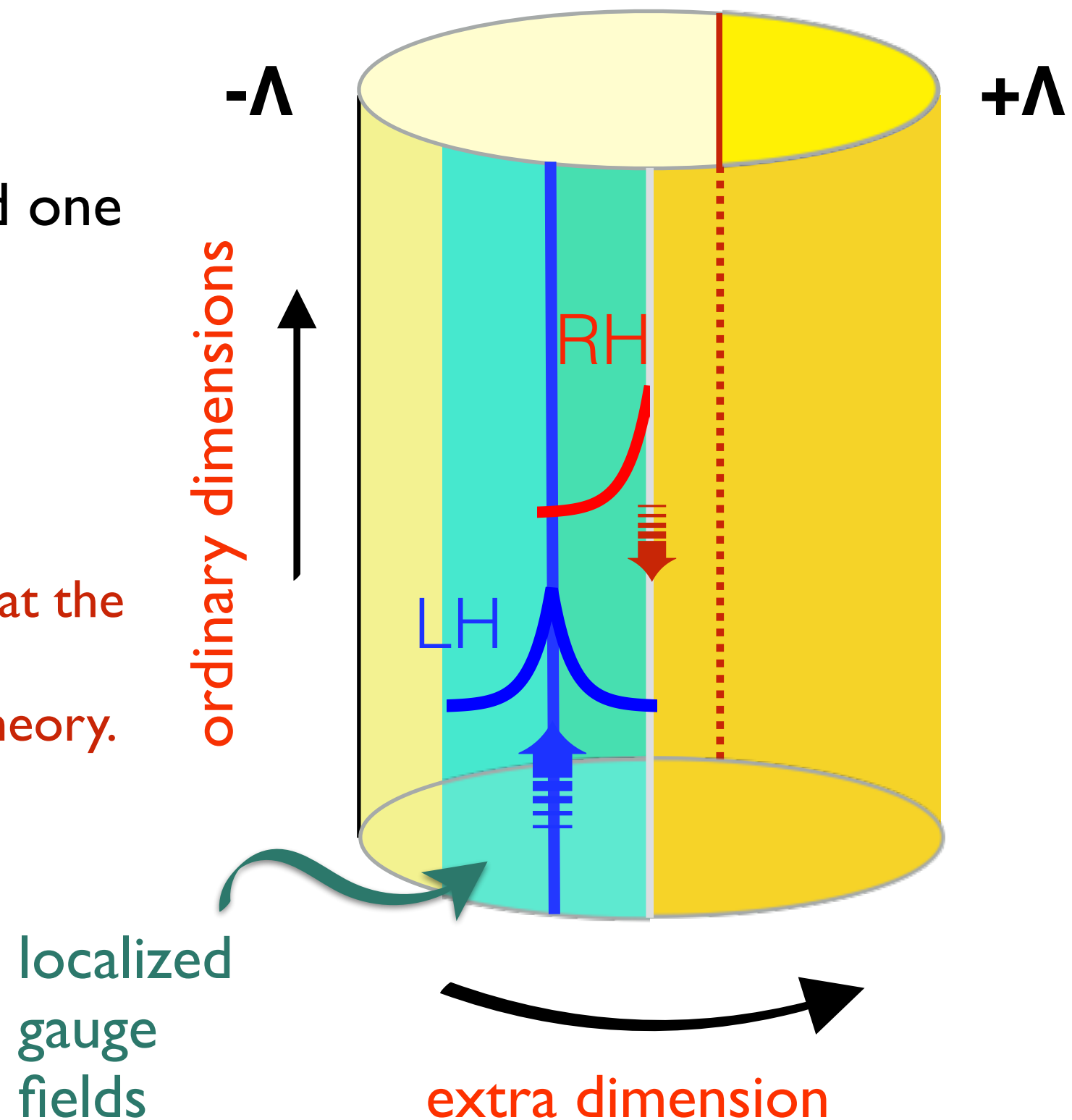
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Localize the gauge fields around one of the defects?

Doesn't work.

Typically find new RH mode appears at the boundary of the SSB phase region
⇒ Dirac fermion, vector like gauge theory.

Golterman, Jansen, Vink 1993

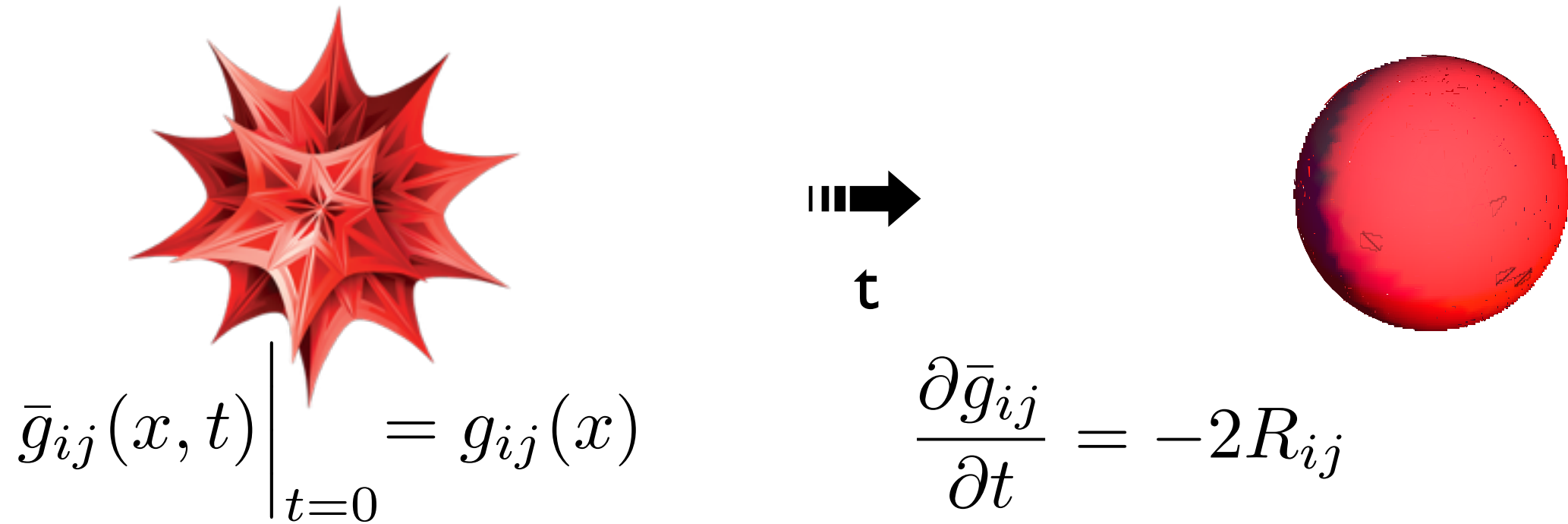


Proposal: “localize” gauge fields for domain wall fermions using *gradient flow*

Technique introduced by mathematicians for smoothing manifolds

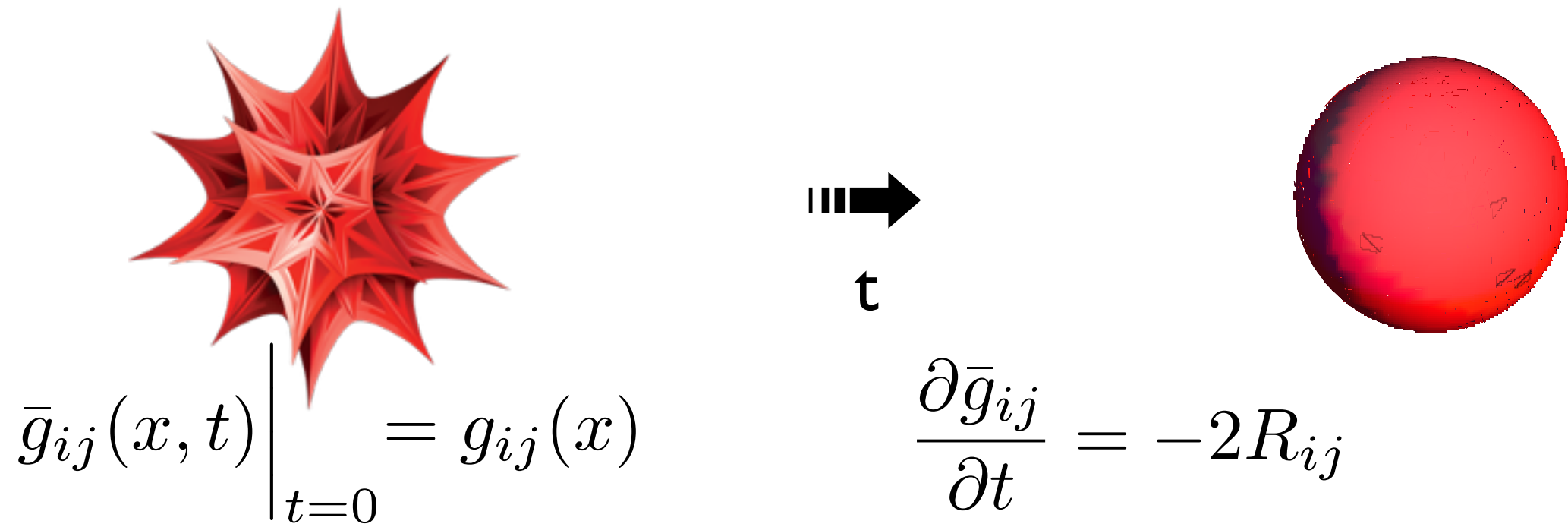
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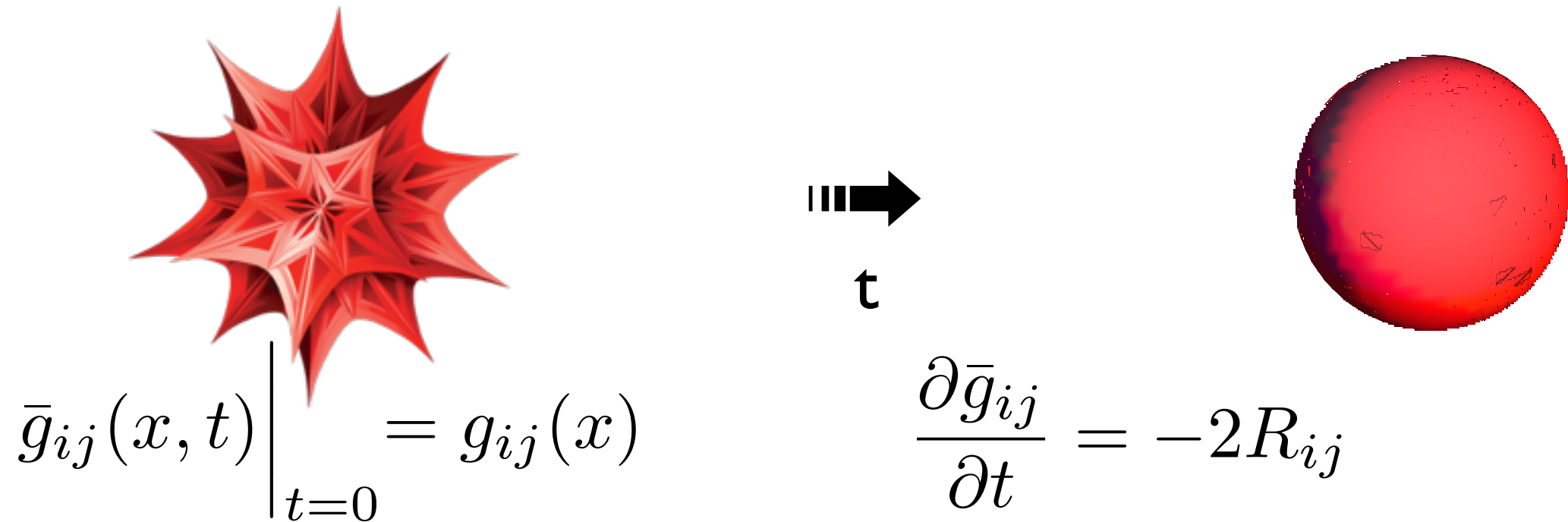
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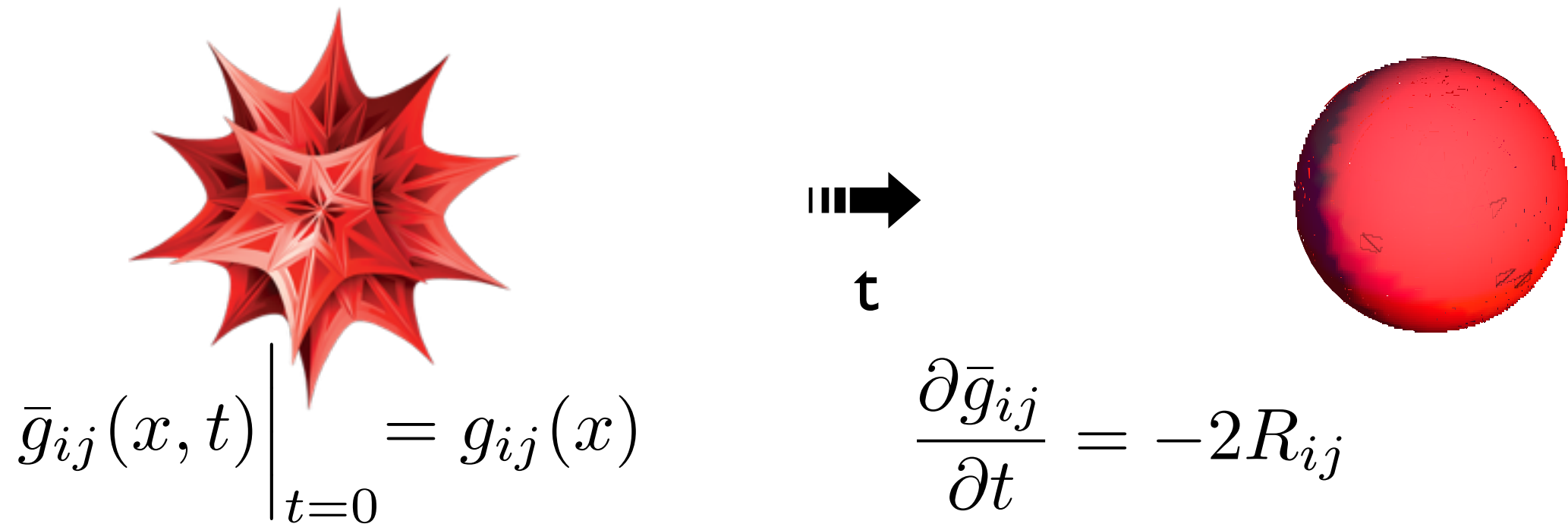
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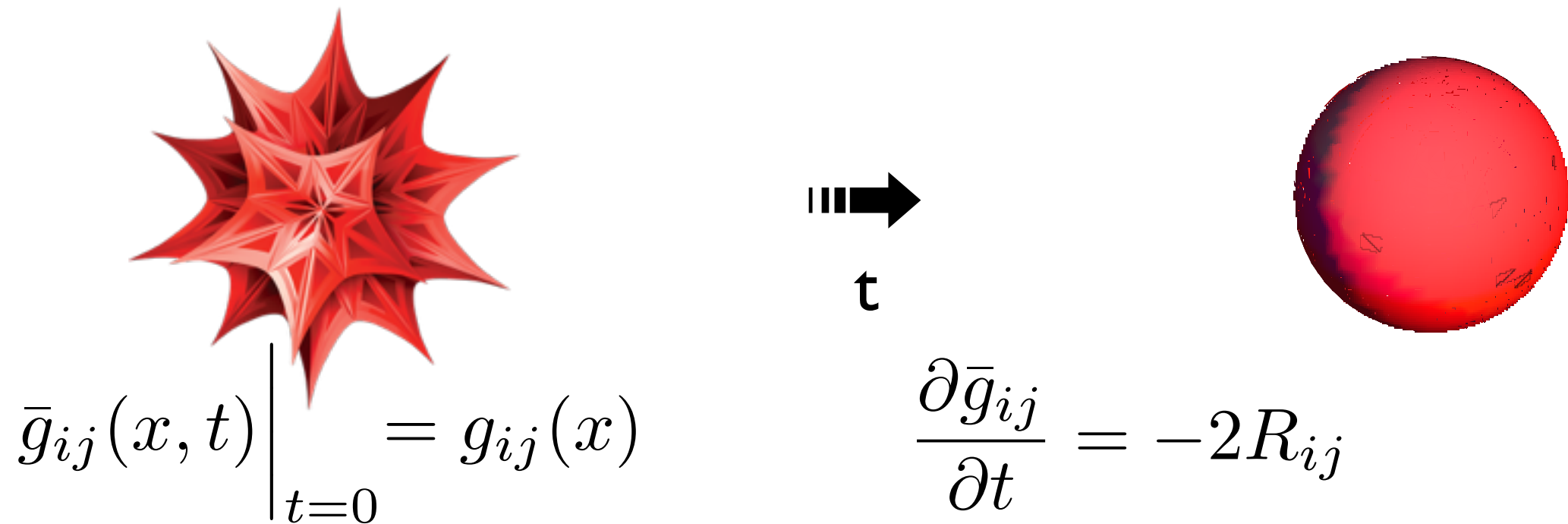
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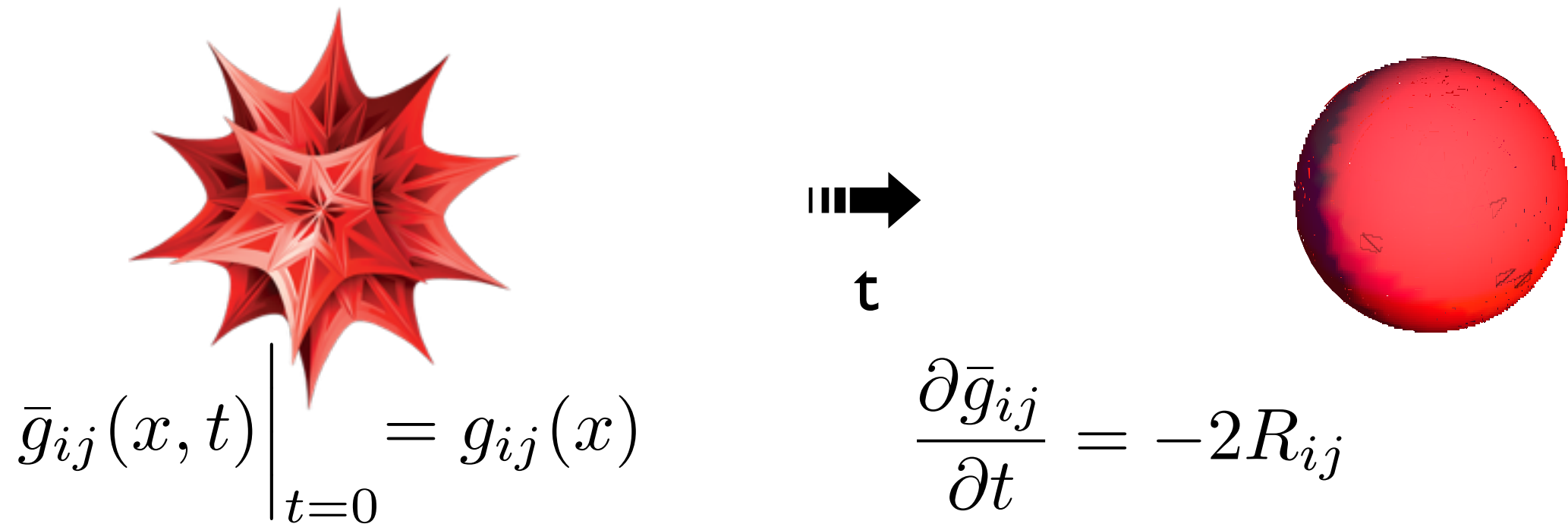
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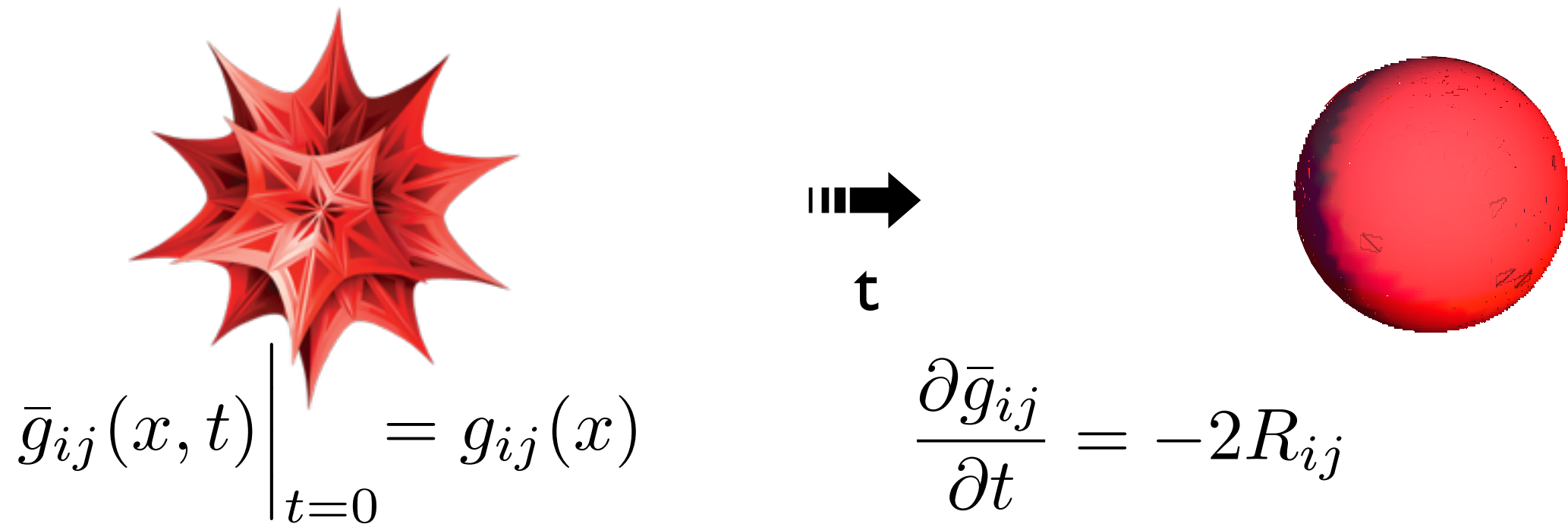
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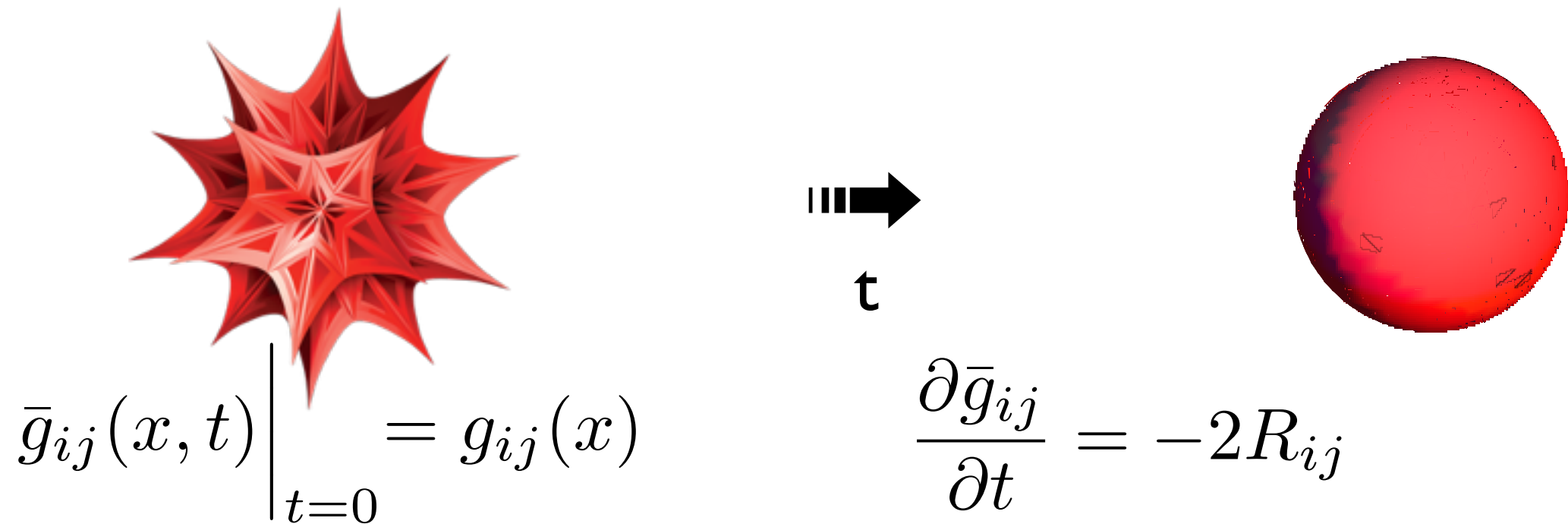
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- **diffeomorphism covariant**

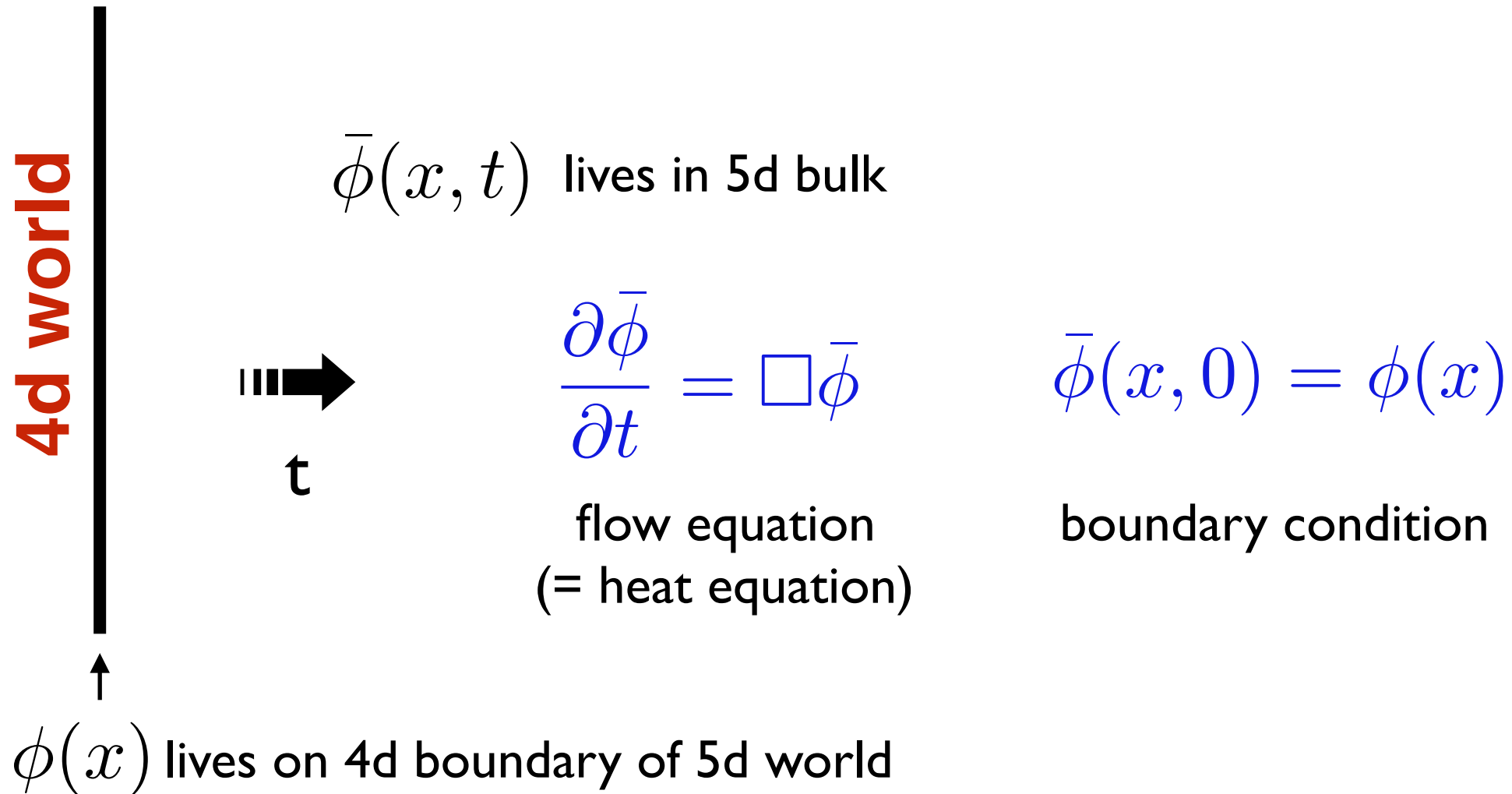
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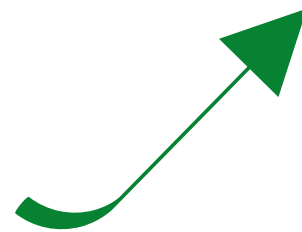
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- **diffeomorphism covariant**
- used by Perelman to prove Poincaré hypothesis

A scalar version of gradient flow

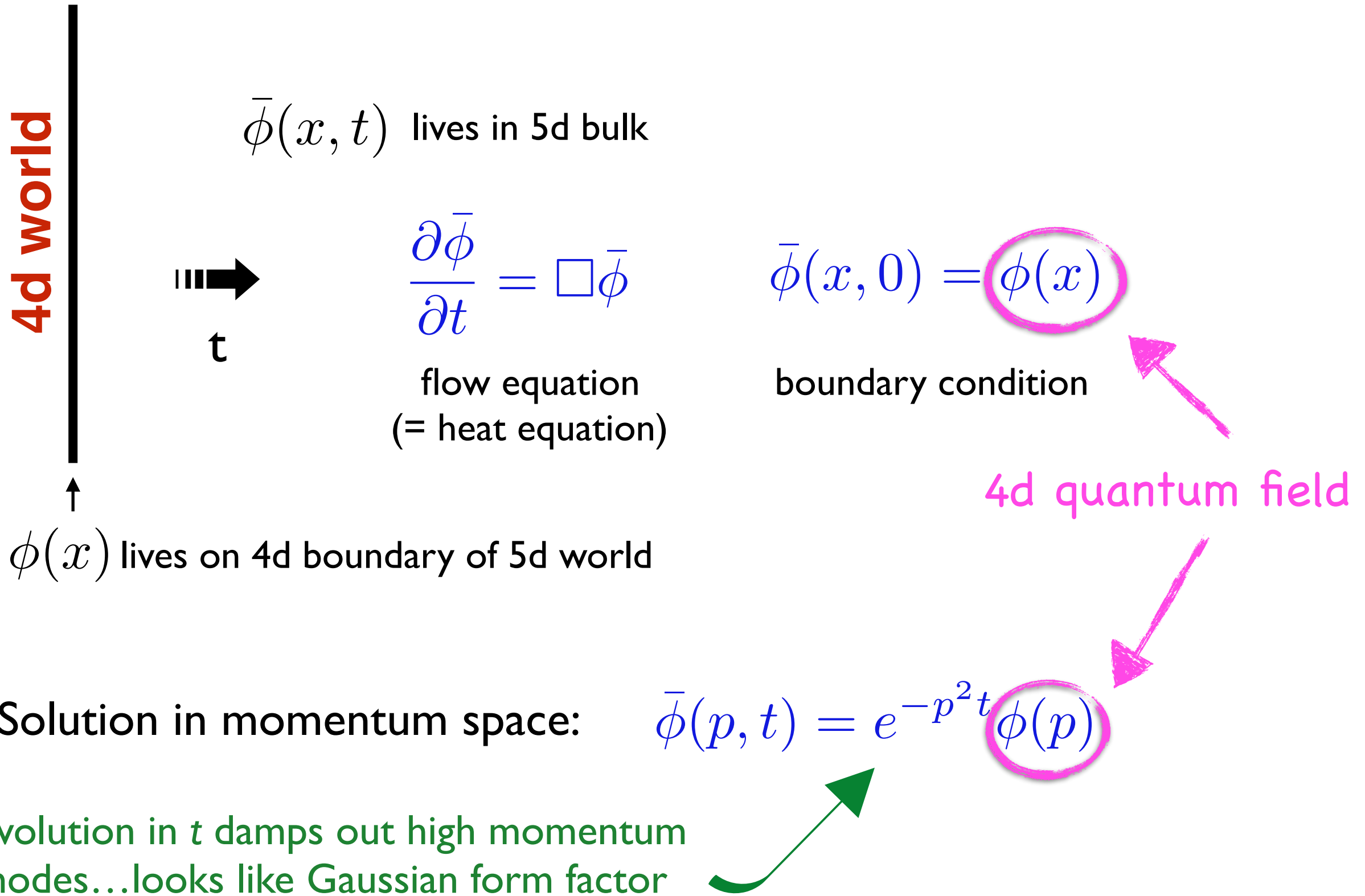


Solution in momentum space: $\bar{\phi}(p, t) = e^{-p^2 t} \phi(p)$

Evolution in t damps out high momentum modes...looks like Gaussian form factor

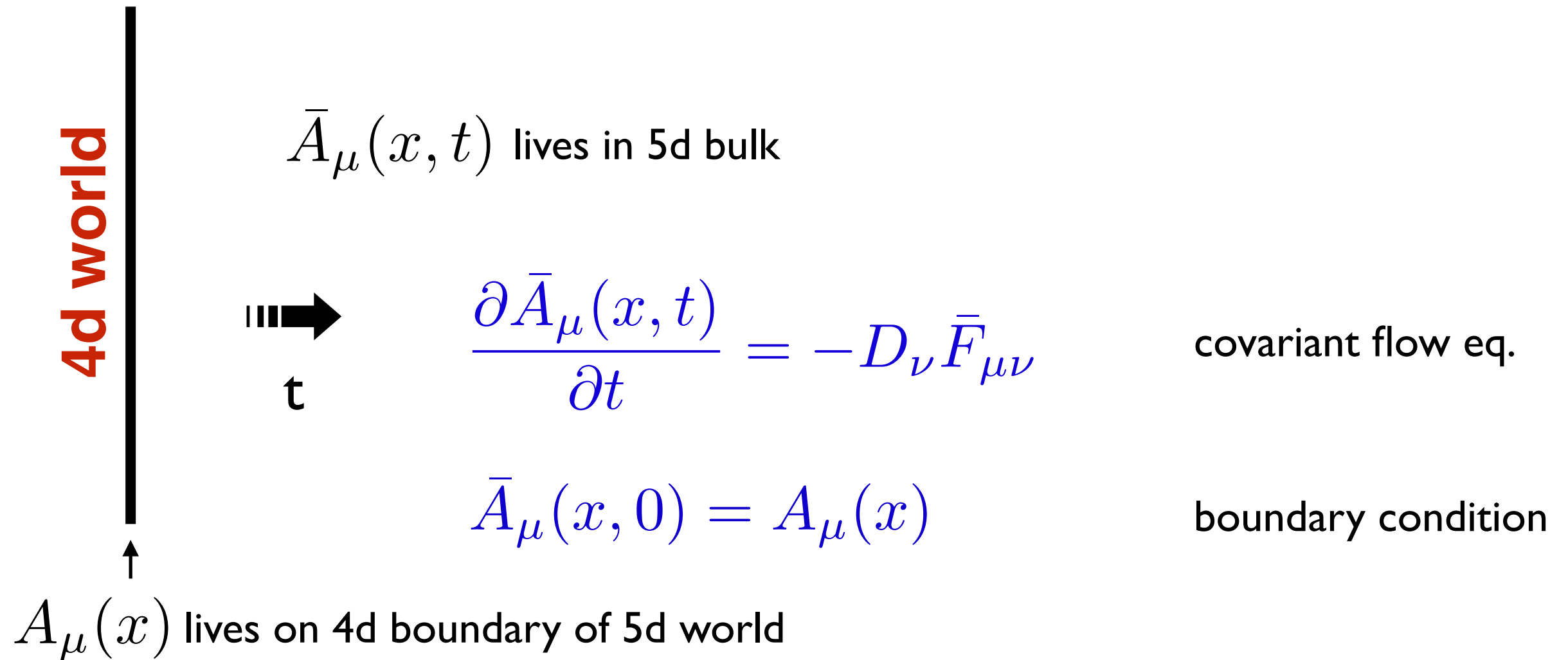


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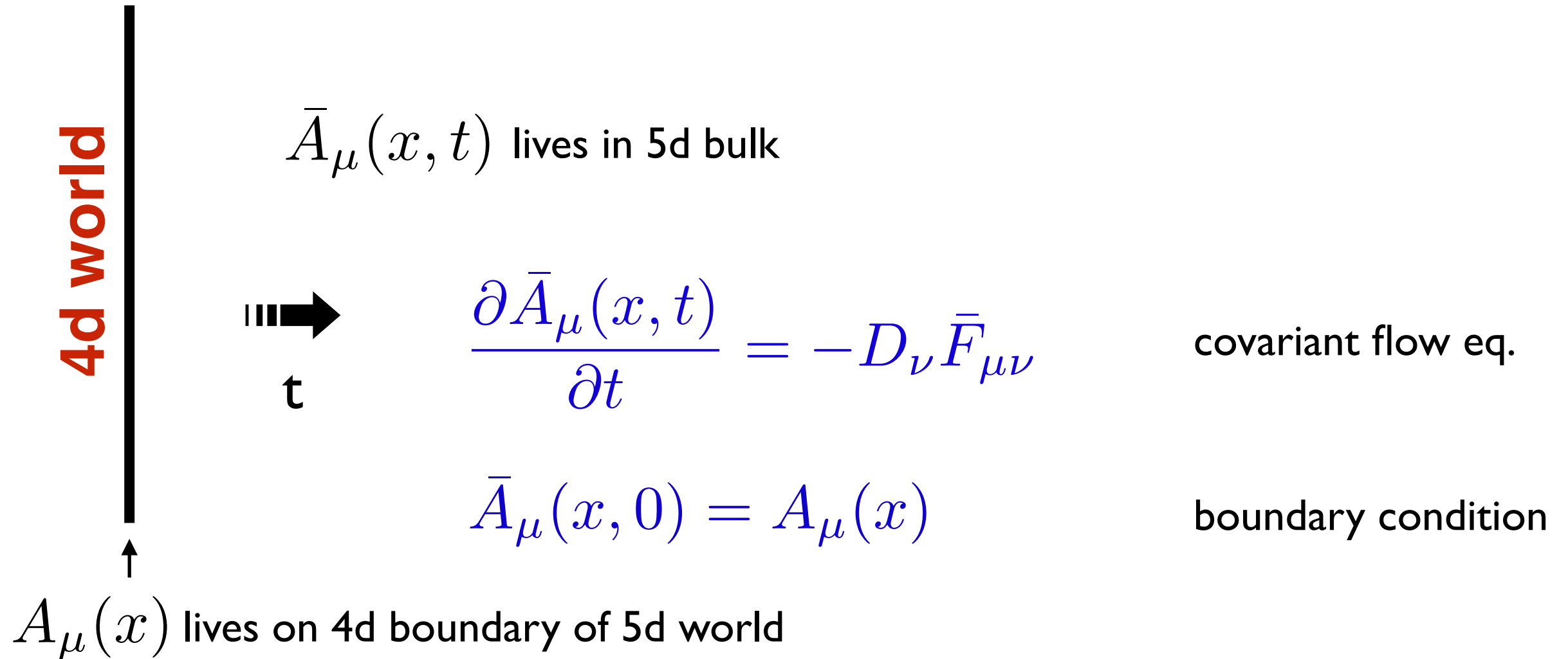


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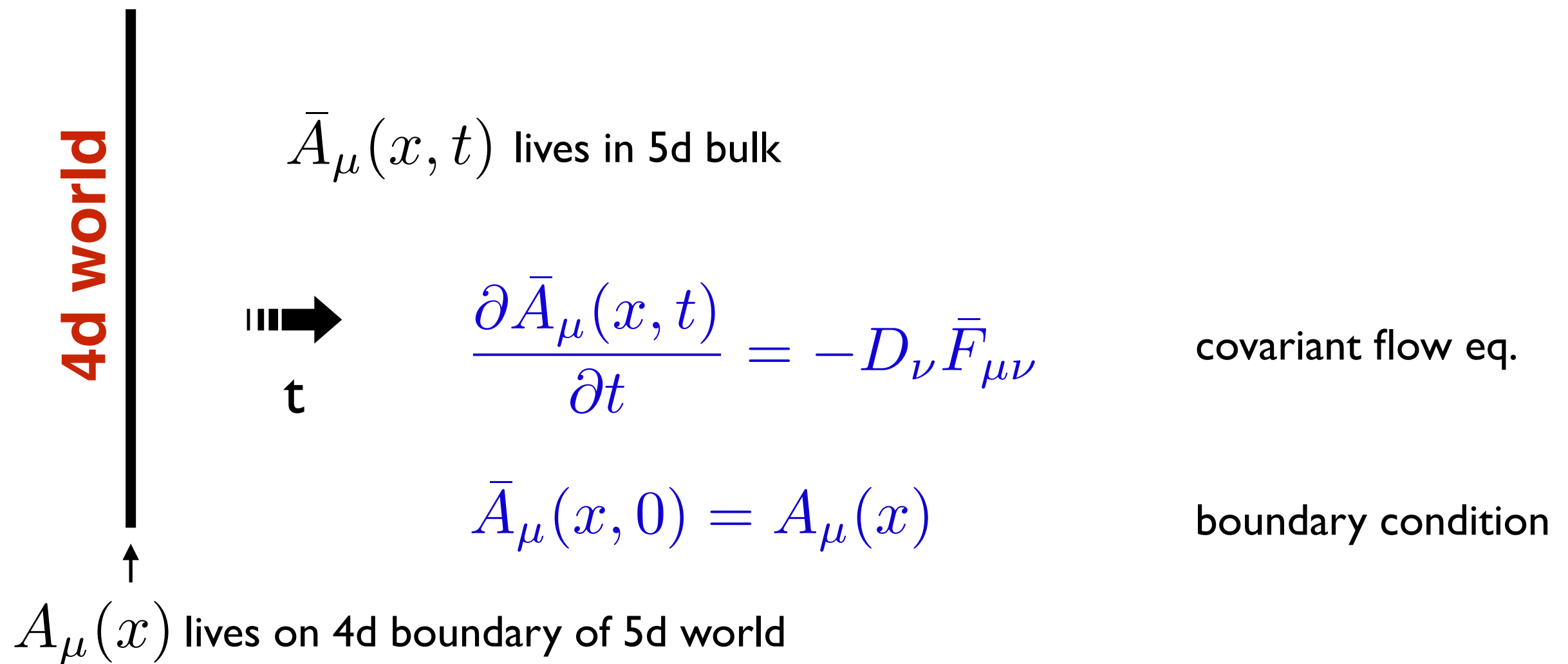
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2d/3d U(1)
example:

$$A_\mu \equiv \partial_\mu \omega + \epsilon_{\mu\nu} \partial_\nu \lambda \quad \Rightarrow \quad \partial_t \bar{\omega} = 0, \quad \partial_t \bar{\lambda} = \square \bar{\lambda}$$

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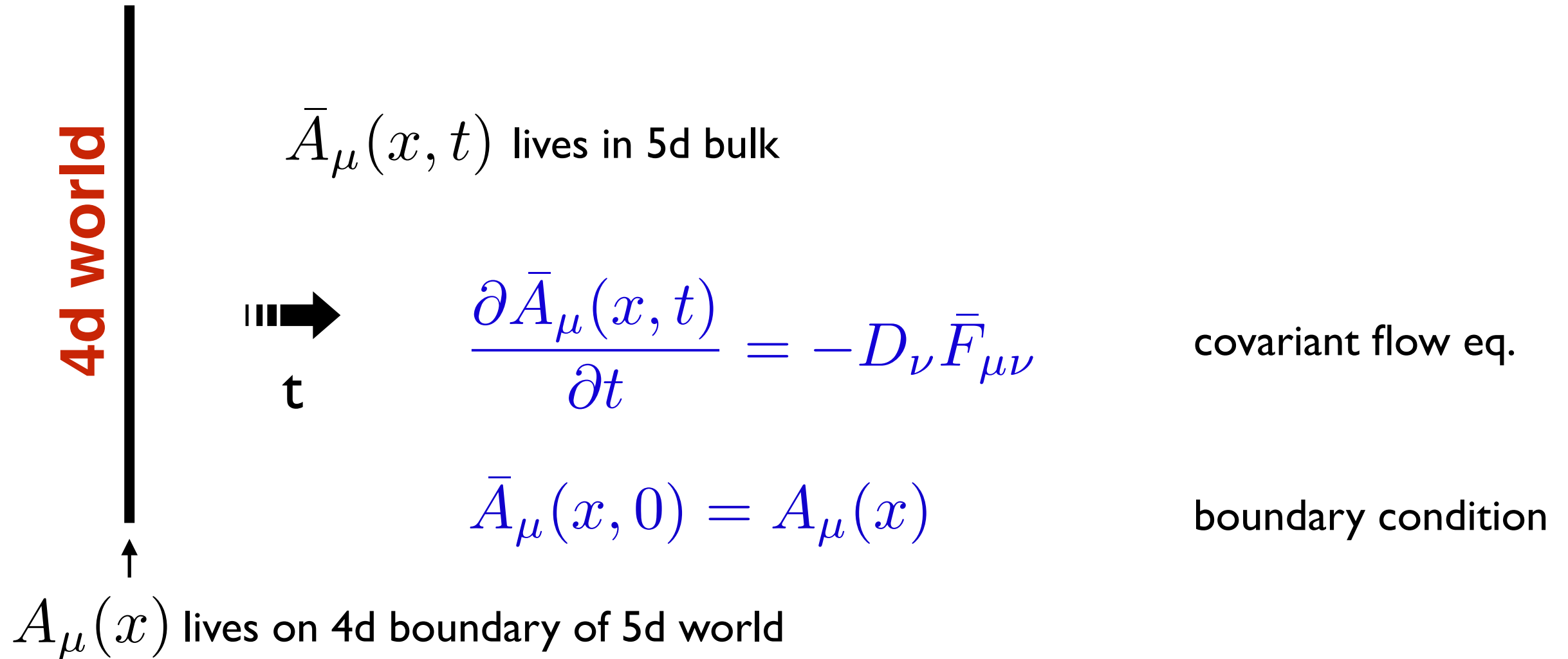


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Evolution in t damps out high momentum modes in physical degree of freedom only

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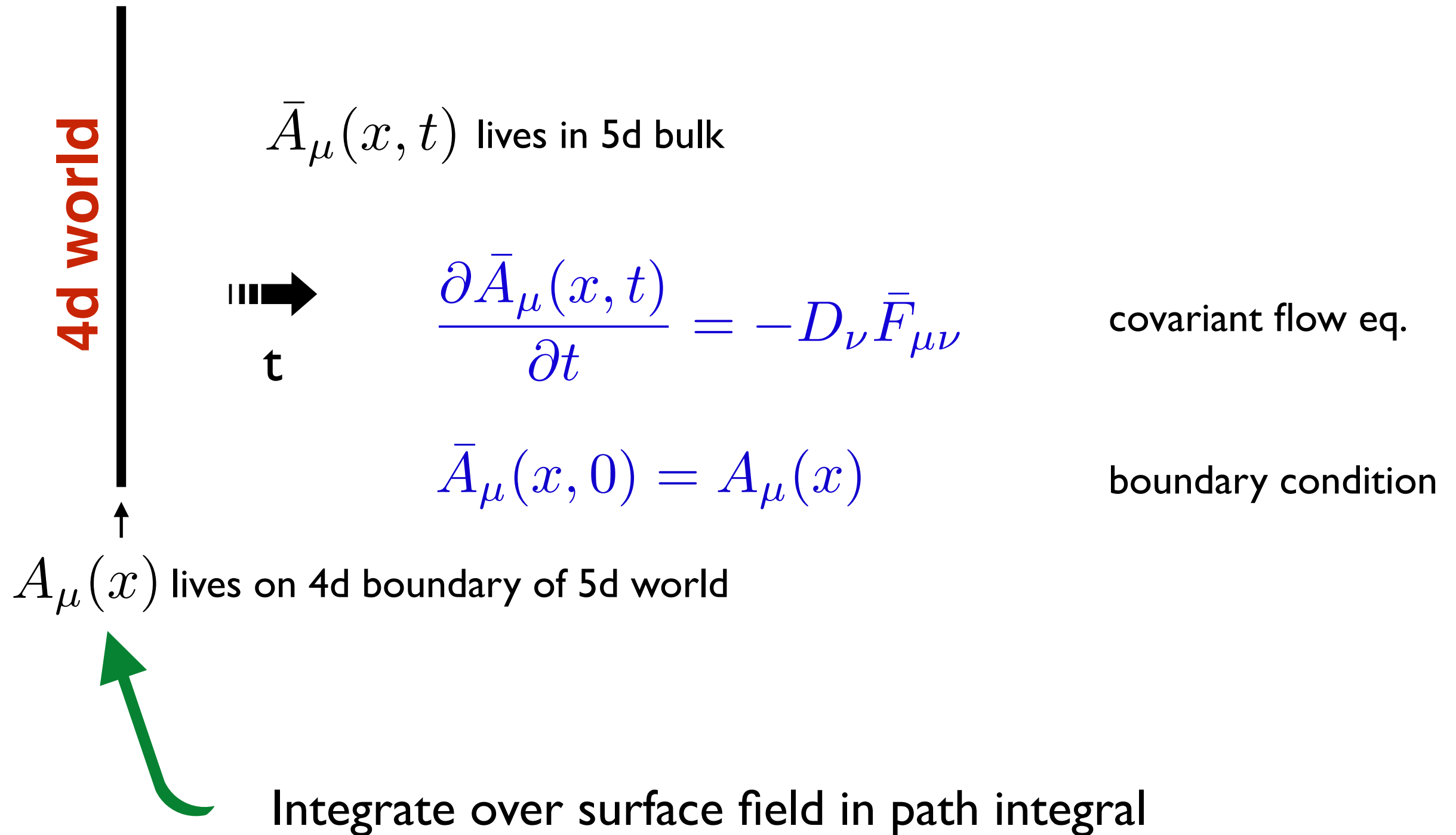
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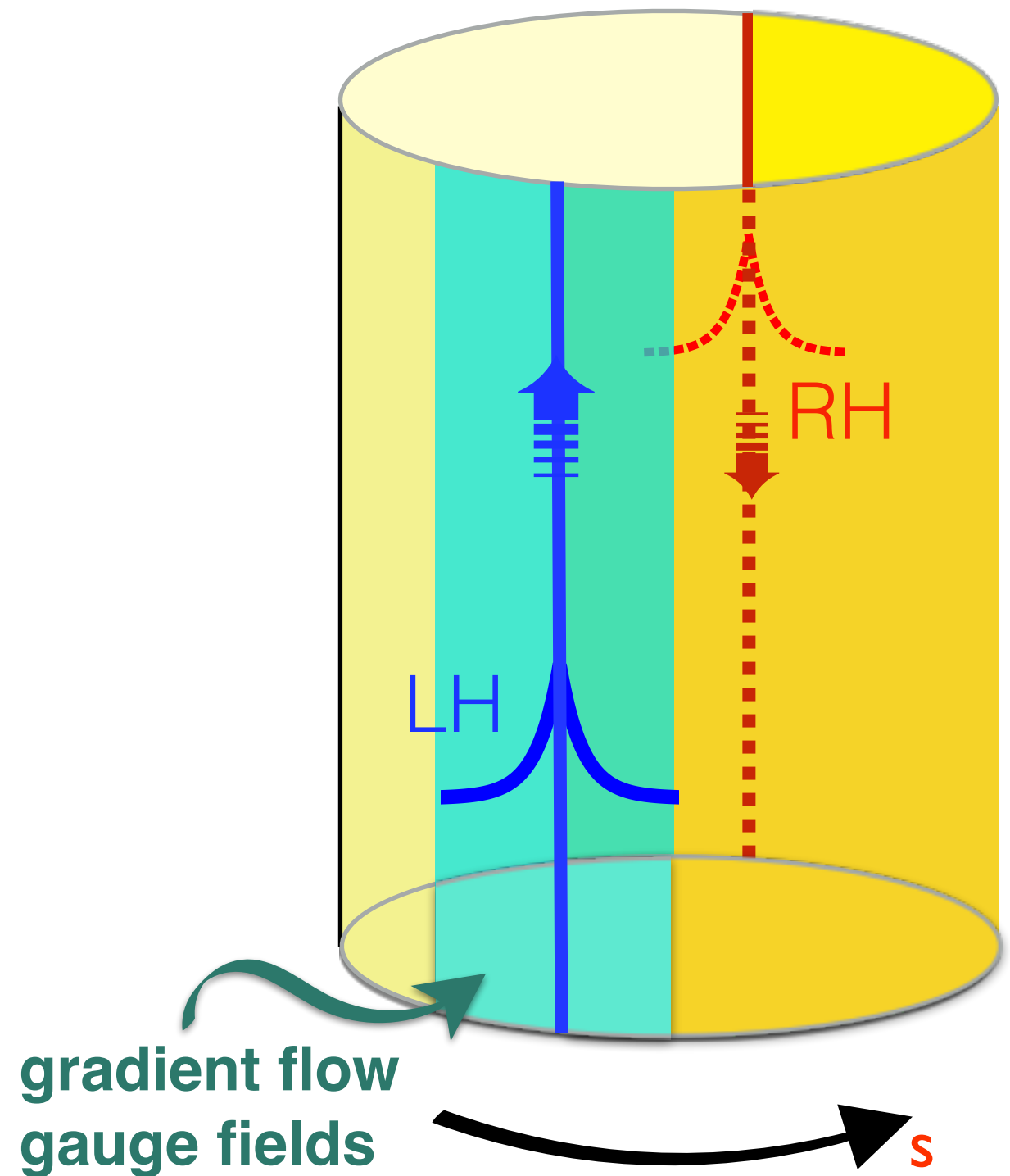
This will allow $\lambda(p)$ to be localized while maintaining gauge invariance

A quantum version of gradient flow for gauge fields (Lüscher, 2010)



Combining gradient flow gauge fields with domain wall fermions:

- quantum gauge field $A_\mu(x)$ lives at defect at $s=0$ where LH fermions live
- gauge field $A_\mu(x,s)$ defined as solution to gradient flow equation with BC:
 $A_\mu(x,0) = A_\mu(x)$
- flow equation is symmetric on both sides of defect
- RH mirror fermions behave as if with very soft form factor... "Fluff"... and decouple from gauge bosons*
- gauge invariance maintained



Concrete proposal for fermion “determinant” for chiral gauge theories:

$$\langle F(A) \rangle = \frac{\int [dA_\mu] e^{-S(A)} \Delta(A) F(A)}{\int [dA_\mu] e^{-S(A)} \Delta(A)} ,$$

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One factor for
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5d Dirac operator
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DWF, sign of
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Constant mass,
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The diagram shows the equation $\Delta(A) = \prod_i \frac{\det(\not{D} - \Lambda_i \epsilon(s))}{\det(\not{D} - \Lambda_i)}$ enclosed in a light blue cloud. Green arrows point from descriptive text to parts of the equation: one to the product symbol, one to the numerator determinant, one to the denominator determinant, and one to the Λ_i term in the denominator.

$\Delta(A) = \prod_i \frac{\det(\not{D} - \Lambda_i \epsilon(s))}{\det(\not{D} - \Lambda_i)}$

- One factor for each 4d fermion
- 5d Dirac operator with flowed gauge field
- DWF, sign of Λ determines chirality
- Constant mass, cancels bulk fermion contributions

Analytic, gauge invariant

What happens for anomalous theory?

How does this proposal distinguish between anomalous and anomaly-free fermion representations of gauge symmetry?

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Anomaly-free representation of LH chiral modes:

- Coefficient of bulk CS term cancels between species, theory looks fine
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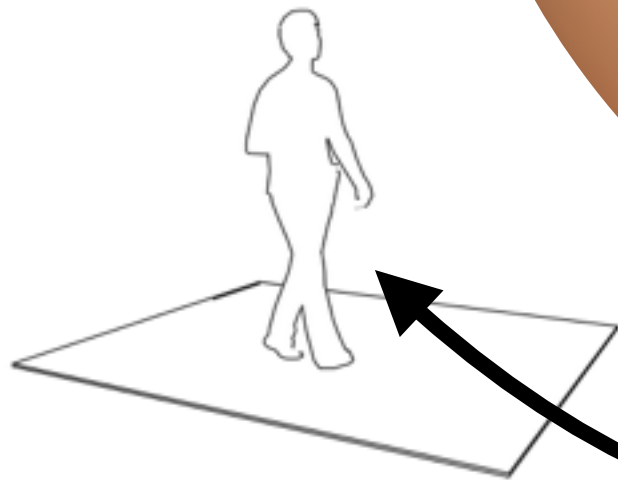
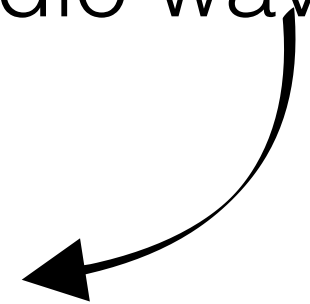
Anomalous representation of LH chiral modes:

- CS operator has nonzero coefficient
- With gradient flowed gauge fields this is not a local operator
- So one gets a gauge invariant theory, but it is not 4d...lost charges are “banked” in the extra dimension

Summary to this point:

- Proposal for measure for chiral gauge theories
- Involves marriage of domain wall fermions and gradient flow
- Manifestly gauge invariant
- Theory only looks local when fermion representation is anomaly free
- Involves mirror fermions with soft form factors that decouple... “Fluff”
- If fluff exists in the real world, could coupled to Ricci-flowed gravitational field and have non-canonical gravitational interactions

- Mirror top quark (fluff)
- mass = 170 GeV
- couples only to radio waves

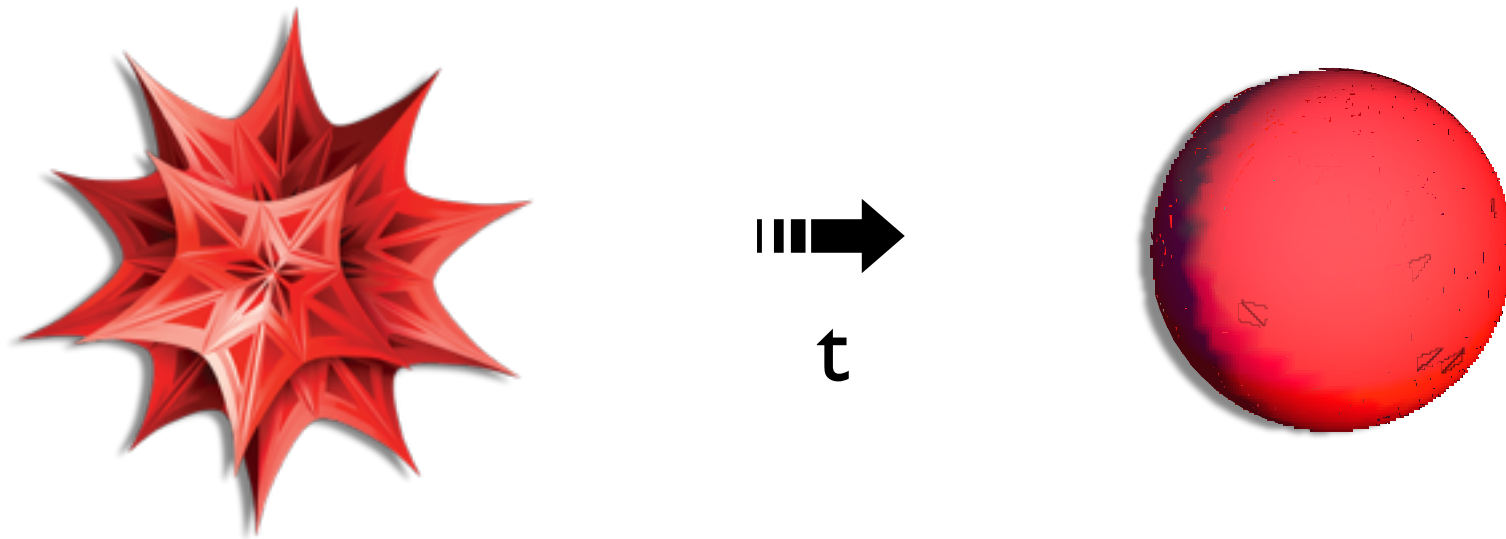


- lattice gauge theorist

The whole story? No!

Flow equation:
$$\frac{\partial \bar{A}_\mu(x, t)}{\partial t} = -D_\nu \bar{F}_{\mu\nu}$$

*Rough configurations
tend to flow to smooth
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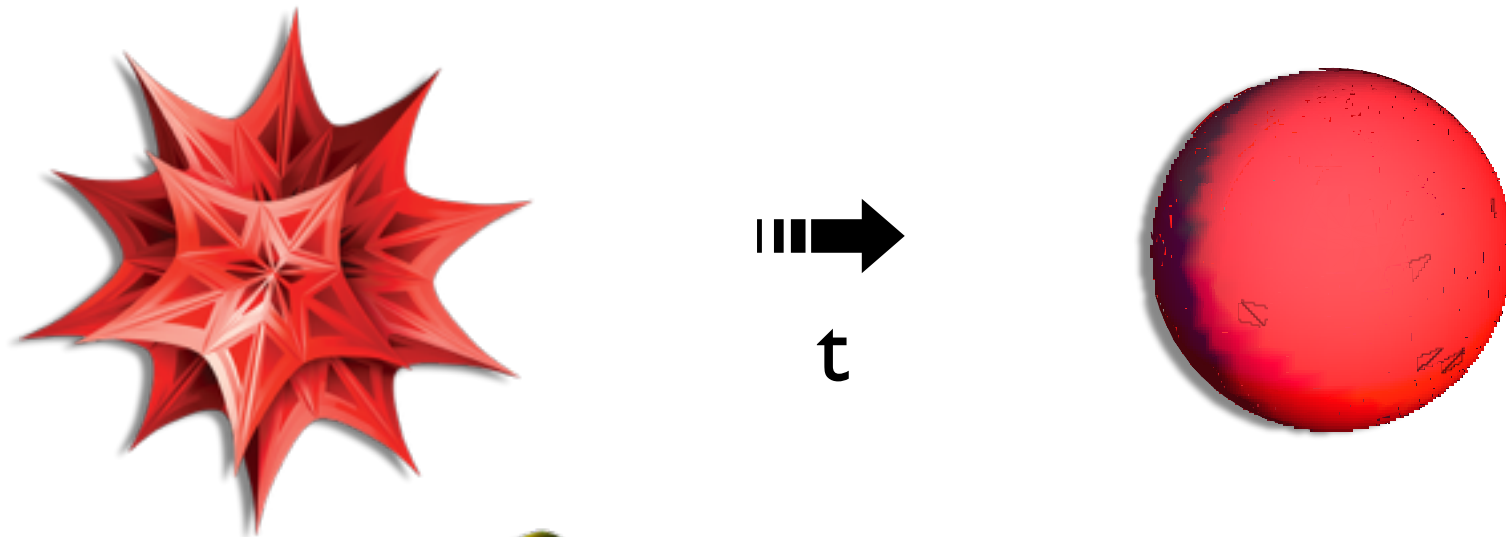


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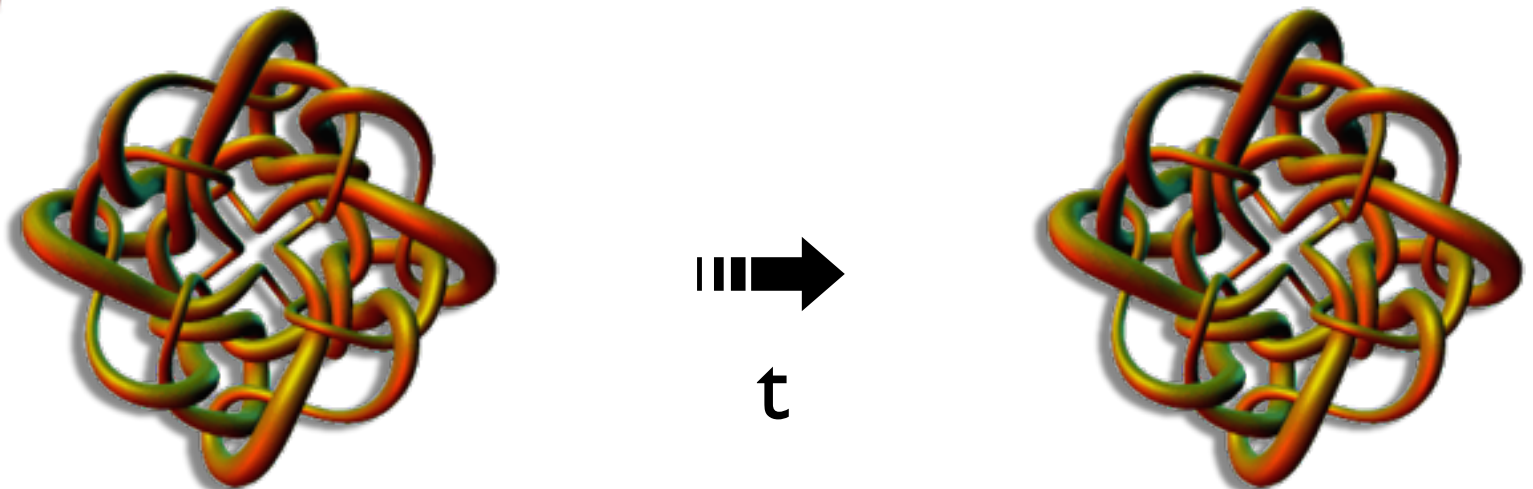
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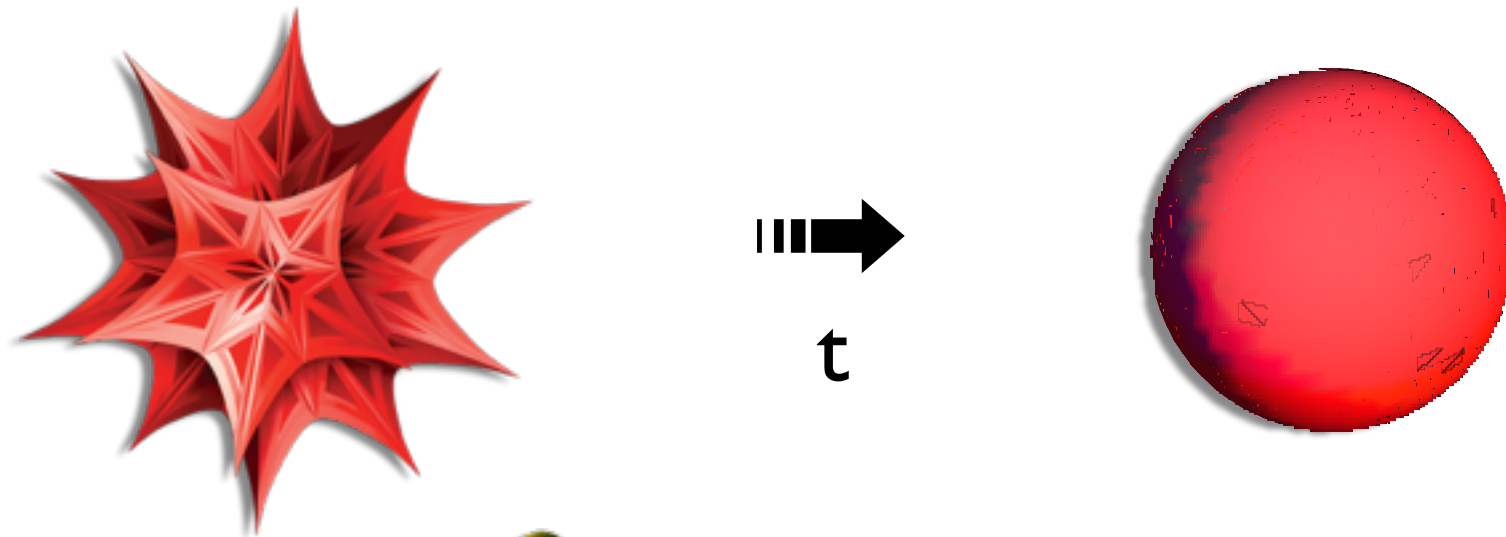
*But solutions to the Euclidian
equations of motion
(in particular, instantons)
do not flow at all*



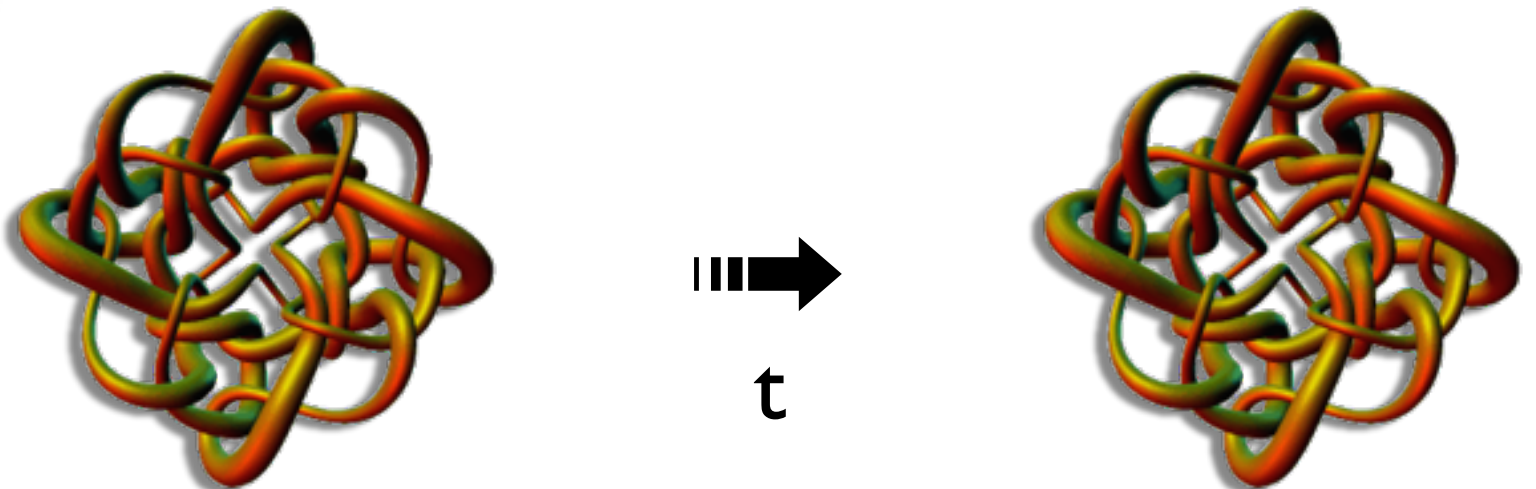
The whole story? No!

Flow equation:
$$\frac{\partial \bar{A}_\mu(x, t)}{\partial t} = -D_\nu \bar{F}_{\mu\nu}$$

*Rough configurations
tend to flow to smooth
ones*

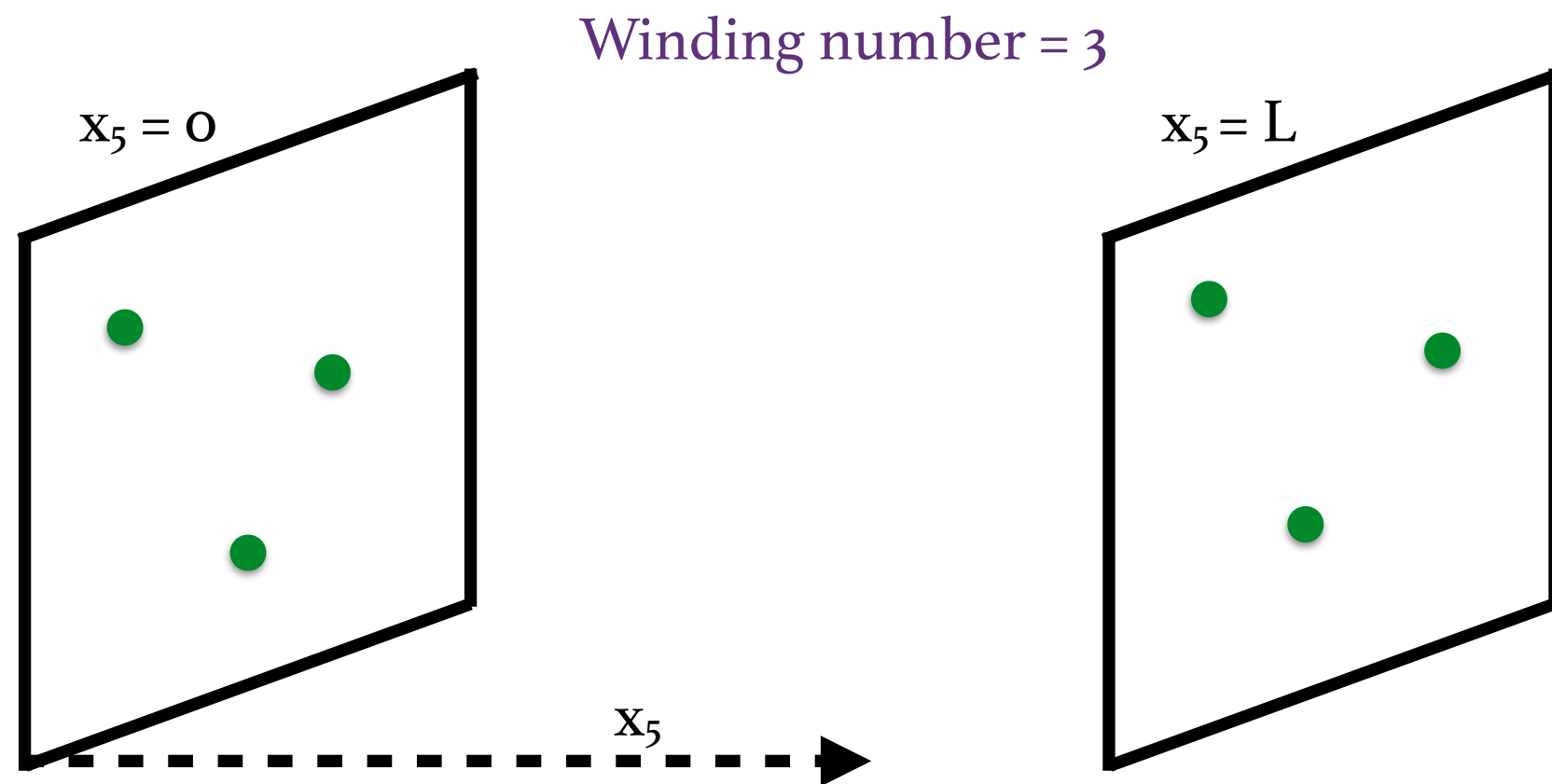


*But solutions to the Euclidian
equations of motion
(in particular, instantons)
do not flow at all*

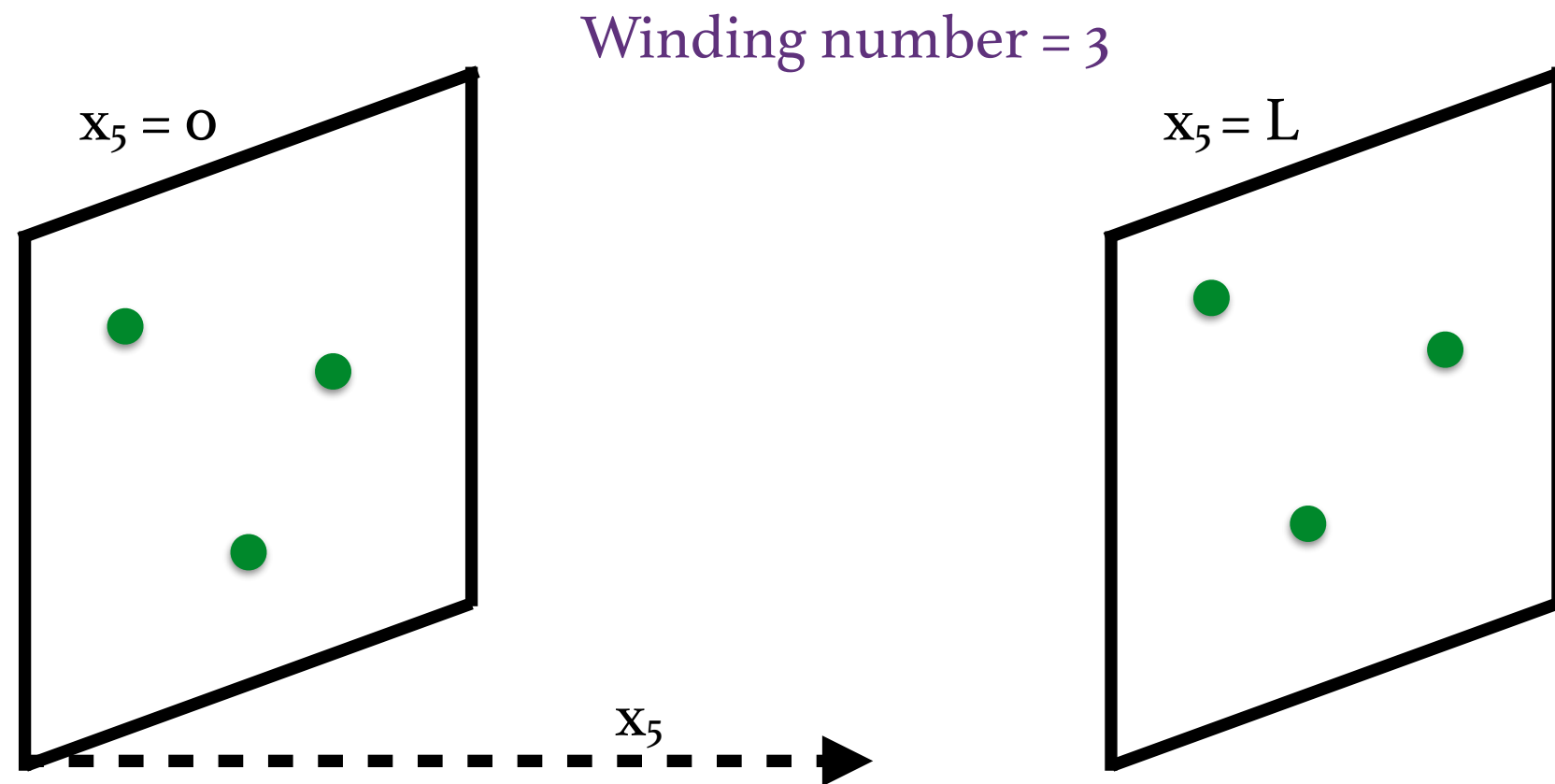


Fluff will not decouple from the topological features of the chiral gauge theory

Topological Gauge Configurations - Weak Coupling



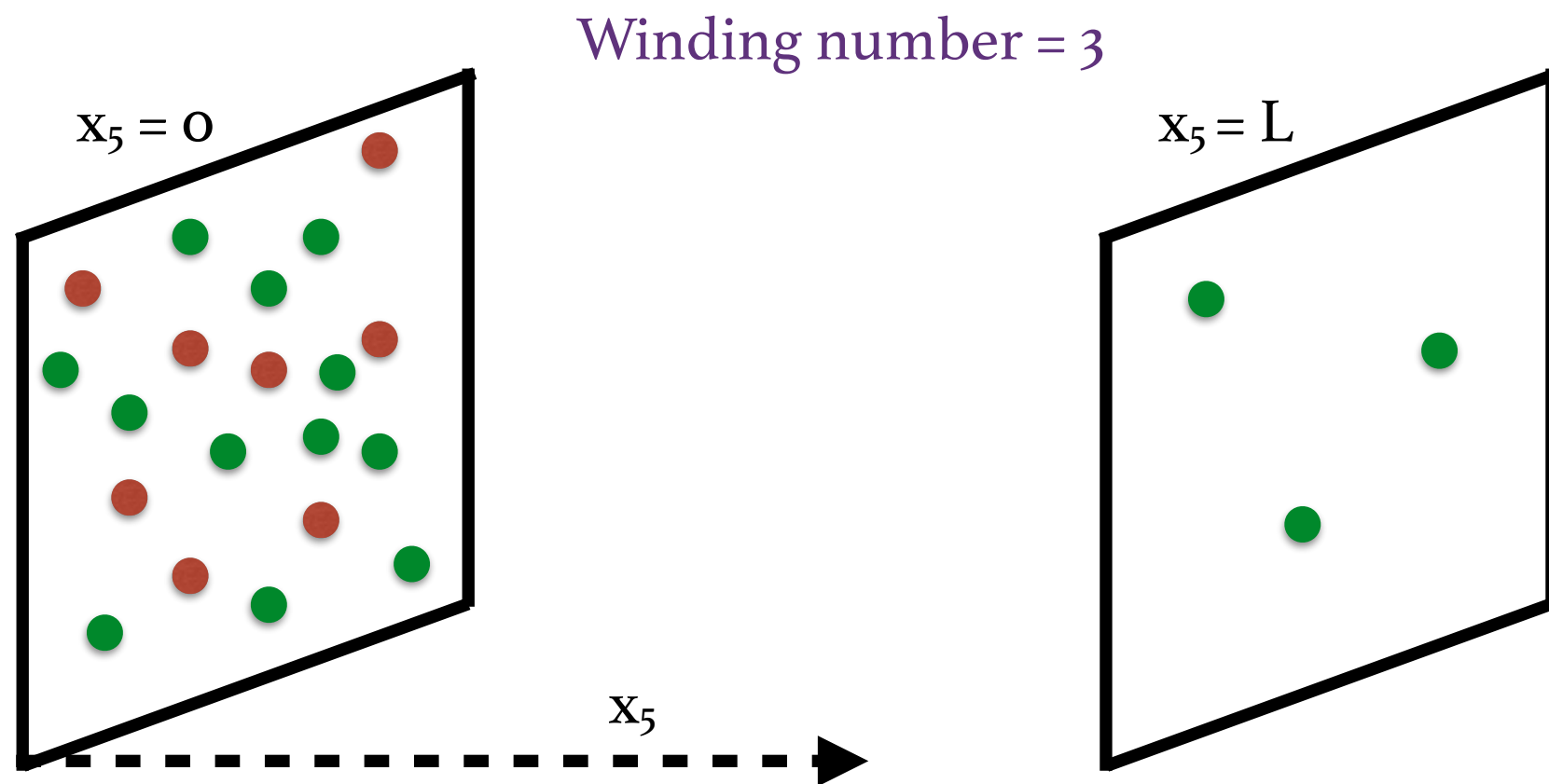
Topological Gauge Configurations - Weak Coupling



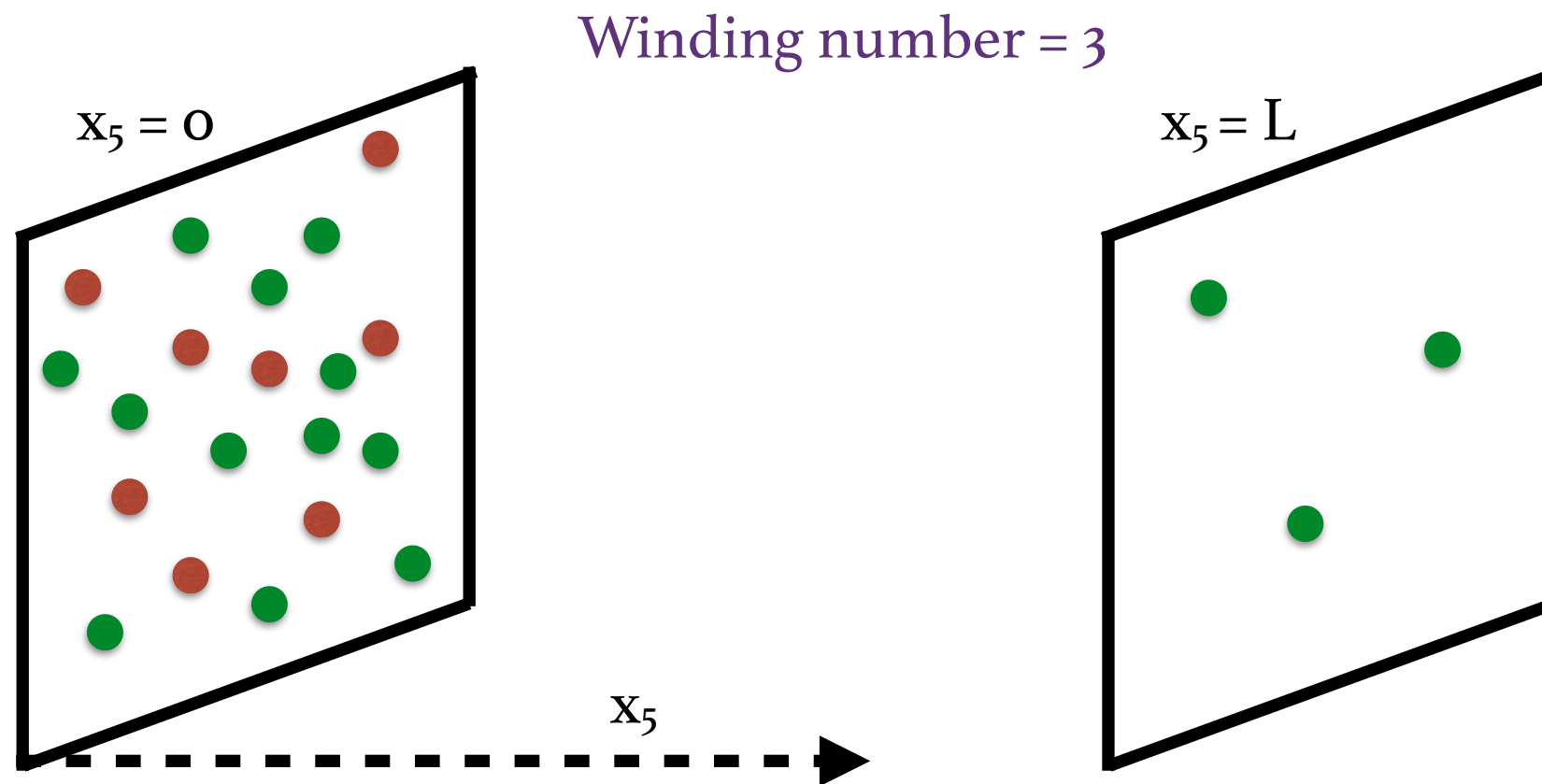
At weak coupling, minimal instanton contribution for given winding # is most important

- Flow does not affect location of instantons
- Correlation between location of instantons on the two boundaries allows for exchange of energy/momentum through 't Hooft vertices
- Weak coupling \Rightarrow exponentially suppressed process

Topological Gauge Configurations - Strong Coupling



Topological Gauge Configurations - Strong Coupling



At strong coupling, need to include instanton-anti instanton pairs

- I-A pairs DO NOT satisfy equations of motion
- If flow for sufficiently large L , all pairs will annihilate
- Expect little correlation between location of fermion zero modes on the two boundaries
- Expect standard fermions and Fluff to interact via bi-local 't Hooft vertices but not exchange energy/momentum?

Conclusions:

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- Motivated by a serious unsolved problem in QFT